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* Publication of closing discussion pending.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

THE HYDRAULIC JUMP IN SLOPING CHANNELS

By Carl E. Kindsvater,[1] Jun. Am. Soc. C. E.

Synopsis

Common forms of the hydraulic jump in sloping channels have been classified into three general cases, and an analysis is presented which leads to a practical method of computing the dimensions of the jump. Satisfactory agreement between analysis and experiment was obtained from laboratory tests on a channel with a 1 on 6 sloping floor. Conclusions drawn from this investigation indicate that experiments on other slopes might eventually yield a satisfactory treatment for hydraulic jumps on any slope within the practical range.

Introduction

Historical Résumé.—The inability to predict the occurrence or to compute the essential dimensions of the hydraulic jump in sloping channels has been a source of difficulty in hydraulic design.[2] Sufficient verification data now exist to substantiate the application of the momentum principle, derived from Newton's second Law of Motion, to the hydraulic jump in open or enclosed horizontal conduits of any practical cross-sectional shape. To apply the same principle to an analysis of the hydraulic jump in sloping channels, however, it is necessary to obtain floor pressures and certain elements of the jump profile from laboratory experiment.

In their early work[3] on the hydraulic jump, Bidone, Bazin, Bélanger, and Boussinesq were unable to verify their analyses largely because they neglected components of pressure on the sloping floor. Others, more recently, have made the same error. In 1934, the late D. L. Yarnell, M. Am. Soc. C. E., of the U. S. Department of Agriculture, Division of Drainage, attempted an analysis in which floor pressures under the jump were evaluated from an assumed straight-line profile from the beginning of the jump to the end of the roller. This approximation was not sufficiently accurate to give satisfactory results. In

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **April 1, 1943.**

[1] Asst. Hydr. Engr., Flood Control Section, TVA, Knoxville, Tenn.

[2] "Hydraulic Tests on the Spillway of the Madden Dam," by Richard R. Randolph, Jr., *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1086.

[3] "The Hydraulic Jump in Sloped Channels," by B. A. Bakhmeteff and A. E. Matzke, *Transactions*, A. S. M. E., Vol. 60, 1938, Paper HYD-60-1, pp. 111–118.

1936, Mr. Yarnell resumed his investigation with a program of comprehensive laboratory tests, which were interrupted by his death on March 9, 1937.

In 1938, B. A. Bakhmeteff, M. Am. Soc. C. E., and A. E. Matzke, Assoc. M. Am. Soc. C. E., published[3] an analysis in which, by introducing a "form coefficient" based on profile measurements, they accounted for the effect of pressures on the sloping floor. Verification of their treatment was limited, however, to a single case of the jump in channels of very flat slope.

The present investigation, based on Mr. Yarnell's 1936 experiments, was undertaken by the Tennessee Valley Authority (TVA) in preliminary studies of spillway design for one of its dam projects.

Classification of the Hydraulic Jump.—The hydraulic jump occurs under many conditions in a great number of related forms. For facility of analysis it is desirable to classify into several general cases the most common forms of the jump in open rectangular conduits. Fig. 1 shows four basic cases selected

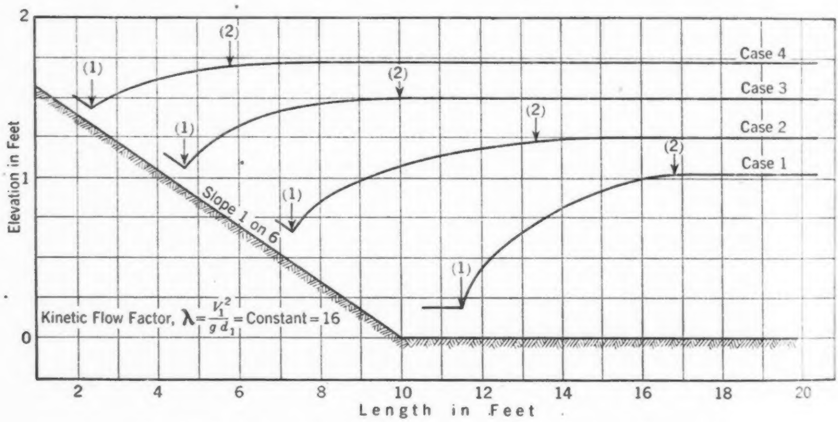


FIG. 1.—BASIC CASES OF THE HYDRAULIC JUMP IN SLOPING CHANNELS

for the present analysis, designated cases 1 to 4 in order of position relative to the sloping floor. Case 1, with the entire roller on the horizontal floor, is the hydraulic jump in horizontal channels. Cases 2, 3, and 4 are common forms of the jump in a sloping channel. Thus, in Case 2 the toe of the roller is on the slope and the end of the roller is on the horizontal floor; in Case 3 the toe of the roller is on the slope and the end is at the junction of the sloping and horizontal floors; in Case 4 the entire roller is on the slope. In each case, the water surface and channel bottom downstream from the jump, as well as the reference axis, are assumed to be horizontal. Messrs. Bakhmeteff and Matzke[3] investigated a fifth case, in which the water surface downstream from the jump was parallel to the slope. This case occurs in some natural channels and in long flumes of flat slope.

NOTATION

The following letter symbols, selected for use in this paper, conform essentially to American Letter Symbols for Hydraulics, prepared by a Committee

of American Standards Association, with Society representation, and approved by the Association in 1942:[4]

A = area of cross section, in square feet;
d = depth of flow, in feet, with appropriate subscripts to denote sections;
f = "function of";
g = acceleration due to gravity, in feet per second per second;
j = subscript denoting "hydraulic jump"; for example, "d_j = depth of hydraulic jump" and "L_j = length of hydraulic jump";
k = coefficient for Case 2 (dimensionless);
L = length along channel, in feet;
n = a subscript denoting "normal"; for example, p_n denotes "normal unit pressure";
P = total pressure, in pounds;
p = unit pressure, in pounds per square foot;
Q = total rate of discharge, in cubic feet per second;
r = radius; as a subscript, r denotes "roller"; for example, L_r = length of roller;
s = embankment or side slopes; as a subscript s denotes sloping floor; for example, L_s = length along a sloping channel floor;
V = average velocity of flow, in feet per second; $V = \frac{Q}{A}$;
y = vertical height of the pressure gradient above the floor, in feet; $\bar{y}$ = average height along a given length of channel;
α = slope angle ($\tan \alpha$ = slope of channel);
γ = unit weight of water, in pounds per cubic foot;
λ = kinetic flow factor, $\frac{(V_1)^{1/2}}{g\, d_1}$;
ϕ = pressure coefficient (dimensionless).

Hydraulic Jump in Horizontal Channels

The theory of the hydraulic jump in horizontal channels, Case 1, has been treated thoroughly by others,[5,6] and repetition is unnecessary except to review the important assumptions involved. Fig. 2 shows the hydraulic jump in a rectangular open channel with a horizontal floor. The body of the jump is delimited by section 1 at the toe of the jump and section 2 at the end of the jump. Since parallel lines of flow are assumed to exist at sections 1 and 2, pressure distribution in both sections is governed by hydrostatic principles. The vertical distances d_1 and d_2 are depths of flow at sections 1 and 2, respectively.

The momentum principle, from Newton's second Law of Motion, states that the time rate of change of momentum between any two sections in a steady flow is equal to the resultant of external forces applied on the mass of fluid between those two sections. Thus, taking the floor of the channel as reference axis, and neglecting the force of boundary friction, the momentum principle

[4] ASA—Z10.2—1942.

[5] "Theory of the Hydraulic Jump and Backwater Curves," by Sherman M. Woodward, *Technical Reports*, The Miami Conservancy District, Pt. III.

[6] "The Hydraulic Jump in Terms of Dynamic Similarity," by Boris A. Bakhmeteff and Arthur E. Matzke, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 630–680.

can be applied to Fig. 2:

$$P_2 - P_1 = \frac{Q\,\gamma}{g}\,(V_1 - V_2)\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

in which P_1 and P_2 are the horizontal forces due to hydrostatic pressure, and V_1 and V_2 are uniform velocities across sections 1 and 2. The pressure-momentum equality in Eq. 1 is illustrated by the superimposed pressure diagrams in Fig. 2.

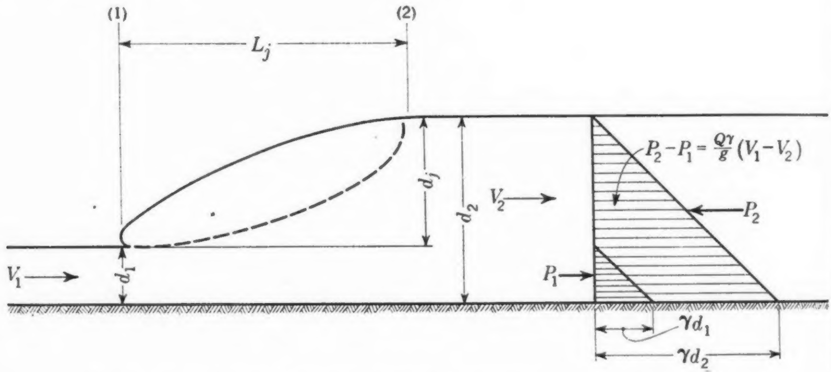


FIG. 2.—CASE 1; THE HYDRAULIC JUMP IN HORIZONTAL RECTANGULAR CHANNELS

Although Eq. 1 is an algebraic expression of the momentum principle, it is not in a form suitable for computation of the dimensions of the hydraulic jump. If the rate of change of momentum is evaluated on the assumption of uniform distribution of the average velocity, Eq. 1 can be written as an equation for d_2 in terms of V_1 and d_1:

$$d_2 = \frac{d_1}{2}\left[\sqrt{\frac{8\,(V_1)^2}{g\,d_1} + 1} - 1\right]\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Messrs. Bakhmeteff and Matzke[6] have shown that the term $\frac{(V_1)^2}{g\,d_1}$, which they have called the kinetic flow factor, is the square of Froude's number, a criterion for dynamic similarity in fluid motion where forces of inertia and gravity predominate. Substituting λ for $\frac{(V_1)^2}{g\,d_1}$, Eq. 2 becomes

$$d_2 = \frac{d_1}{2}\,(\sqrt{8\,\lambda + 1} - 1)\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

THE HYDRAULIC JUMP IN SLOPING CHANNELS

Limitations to Analysis.—There are a number of practical limitations to the application of a generalized analysis of the hydraulic jump in sloping channels. The jump is commonly utilized as a means of dissipating energy below spillways and chutes, frequently as a supplement to impact devices in the stilling basin. Analysis of the latter case is not practical because of the difficulty in evaluating the impact forces on the auxiliary structures.

On very steep slopes on large structures, the problem is often complicated by the entrainment of air in the high-velocity stream before the jump or by the formation of traveling waves on the surface of the stream. In either case, it is practically impossible to compute accurately the momentum or the hydrostatic pressure in a cross section at the toe of the jump.

Hydraulic literature contains many references to differences between the level-floor jump and the jump in sloping channels, with the implication that similar analyses are not feasible for the two cases. In outward appearance, the hydraulic jump on flat slopes is quite similar to the jump in horizontal channels. On steeper slopes, however, the hydraulic jump has been dismissed as an imperfect form of the phenomenon, described as a "drowned nappe" and a "submerged jump." It is possible that these terms apply exclusively to jumps in the Case 2 classification.

Careful observations of several hundred jumps on slopes from 1 on 6 to 1 on 1 indicate that the sloping-floor jump is generally similar to the jump in horizontal channels. Fig. 3 is a mosaic of short-exposure photographs showing a Case 4 jump in a 1 on 6 sloping channel. Fig. 4 shows a Case 2 jump in a 1 on 3 channel. In both exhibits, when the high-velocity stream before the jump impinges on the tailwater, the impact produces a turbulent circulation of "white water" throughout the body of the jump. Immediately downstream from the toe of the jump, the high-velocity stream begins to expand, and the curve of

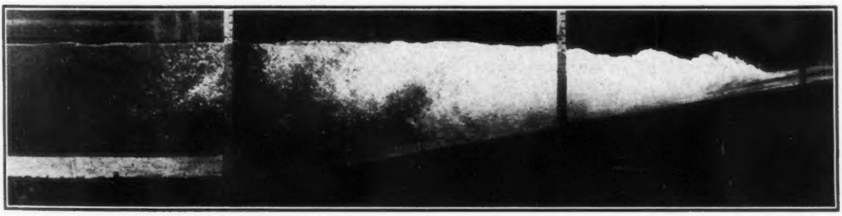
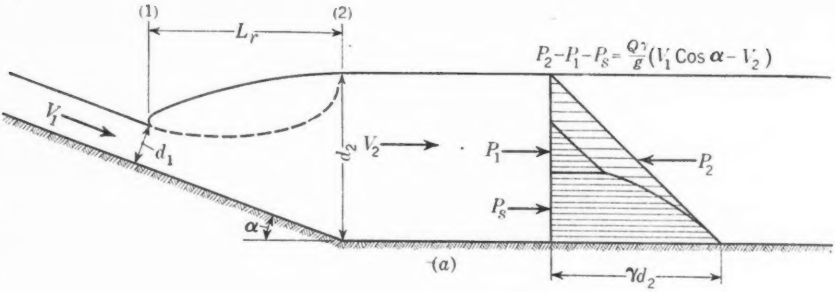


FIG. 3.—CASE 3; HYDRAULIC JUMP IN A 1 ON 6 SLOPING CHANNEL; KINETIC FLOW FACTOR $\lambda = 15.7$

FIG. 4.—CASE 2; HYDRAULIC JUMP IN A 1 ON 3 SLOPING CHANNEL; KINETIC FLOW FACTOR $\lambda = 29.7$

expansion is apparently similar for all cases. However, as the slope of the channel increases, an increasing length of expansion is required before the curve of the upper boundary of the live stream changes from its initial negative slope to a positive slope, ascending to the tailwater surface. On the other hand, the difference in water-surface elevation between sections 1 and 2 decreases with increasing channel slopes.



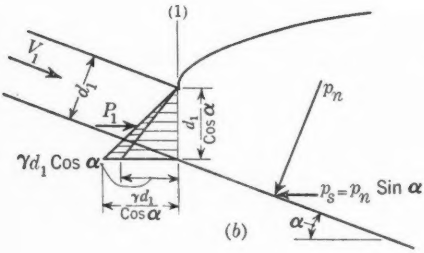


FIG. 5.—CASE 3; HYDRAULIC JUMP IN SLOPING CHANNELS, WITH THE END OF THE ROLLER AT THE JUNCTION OF THE SLOPING AND HORIZONTAL FLOORS

Length of the Jump in Sloping Channels.—Although the length of the jump is not a factor in the analysis leading to Eq. 3 for the level-floor jump, it is a prerequisite to an analysis of the jump in sloping channels. If the channel slope is continuous, as in Case 4, some expansion may occur beyond the end of the jump. Thus, for analytical purposes, it is necessary to know the minimum length of expansion for which the depth of tailwater at section 2 is sufficient to insure the formation of the jump. It is possible to approximate the length of the jump with an assumed constant "safe length," but tailwater depths determined in this manner usually exceed the depth required to produce the jump.

In the present analysis, the length of the jump has been taken as the length of its surface roller. On the water surface, the end of the roller is marked as a narrow zone within which the downstream currents are divided from the upstream currents of the backroller. In the laboratory, the end of the roller was defined reasonably well as the point on the water surface from which confetti particles would float both upstream and downstream. Prolonged observations indicated that the end of the roller was the only point on the surface of the water near the end of the jump which could be determined conveniently and consistently by different observers. Experiments on a 1 on 6 slope showed that the depth of tailwater at the end of the roller was sufficient to produce a jump on the slope.

Since, as in Case 4 jumps, a slight expansion due to the recovery of velocity head may occur beyond the end of the roller, section 2 is not necessarily the point of maximum elevation on the water-surface profile. Nor is it necessarily

true that the end of the roller marks a section of complete dissipation of destructive bottom velocities. However, experiments on a 1 on 6 slope showed a negligible error due to an assumption of uniform velocity distribution at section 2.

Generalized Analysis.—Case 3, with the end of the roller at the end of the slope, is obviously a primary form of the hydraulic jump in sloping channels, since the depth of flow at section 2 is definitely limited to the depth of tailwater. It is convenient for analysis to derive a general treatment from this case. Fig. 5 shows Case 3 in a channel with slope angle α, in which $V_1 \cos \alpha$ is the horizontal component of the average velocity in section 1 at the toe of the jump, and V_2 is the average velocity in section 2 at the end of the jump. The pressures P_1 and P_2 shown on the pressure diagram in Fig. 5(*a*) are the horizontal forces due to pressure on sections 1 and 2, respectively; and P_s is the horizontal component of the total pressure on the sloping floor between sections 1 and 2.

Neglecting boundary friction, the momentum principle applied to Fig. 5 gives

$$P_2 - P_1 - P_s = \frac{Q\,\gamma}{g}\,(V_1 \cos \alpha - V_2) \ldots\ldots\ldots\ldots\ldots (4)$$

The right-hand side of Eq. 4 is equivalent to the rate of change of the horizontal component of momentum between sections 1 and 2. If the velocity at section 2 is evaluated on an assumption of uniform velocity distribution, $V_2 = \frac{Q}{A_2}$, then for a channel of unit width,

$$V_2 = \frac{V_1\,d_1}{d_2} = \left[\frac{V_1 \cos \alpha \dfrac{d_1}{\cos \alpha}}{d_2}\right] \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Substituting this expression for V_2 in Eq. 4, the rate of change of momentum becomes

$$\frac{Q\,\gamma}{g}\,(V_1 \cos \alpha - V_2) = \frac{\gamma\,(V_1)^2\,d_1 \cos \alpha}{g\,d_2}\left(d_2 - \frac{d_1}{\cos \alpha}\right) \ldots\ldots\ldots (6)$$

If the pressure distribution in section 2 is assumed to follow hydrostatic principles, then P_2 in Eq. 4 becomes, for a channel of unit width,

$$P_2 = \frac{\gamma\,(d_2)^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Referring to Fig. 5(*b*), the equation of equilibrium[7] for section 1 gives

$$P_1 = \left(\frac{d_1}{\cos \alpha}\,\frac{\gamma\,d_1 \cos \alpha}{2}\right) = \frac{\gamma\,(d_1)^2}{2} \ldots\ldots\ldots\ldots\ldots (8)$$

but it is convenient for the present analysis to substitute for Eq. 8 an approximation,

$$P_1 = \frac{\gamma\,(d_1)^2}{2 \cos^2 \alpha} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

[7] "Applied Fluid Mechanics," by Morrough P. O'Brien and George H. Hickox, McGraw-Hill Book Co., Inc., New York, N. Y., p. 293.

As far as the final equation for the hydraulic jump is concerned, the difference between Eqs. 8 and 9 is negligible for slopes as steep as 1 on 2.

The term P_s in Eq. 4 must be evaluated from empirical relations established by laboratory measurements of floor pressures. Since experiments indicate that pressure on the sloping floor is proportional to the weight of the fluid body on the slope, P_s is a function of the dimensions and shape of the jump. In this analysis, it is convenient to express P_s in terms of a dimensionless parameter, ϕ, selected to facilitate algebraic operations:

$$P_s = \phi \left[\gamma \, (d_2)^2 - \frac{\gamma \, (d_1)^2}{\cos^2 \alpha} \right] \tan \alpha \quad \text{(10)}$$

As shown subsequently, ϕ is an experimental coefficient, distinct for each value of the kinetic flow factor and each channel slope.

Eq. 4 may now be evaluated in terms of Eqs. 6, 7, 9, and 10:

$$\frac{\gamma \, (d_2)^2}{2} - \frac{\gamma \, (d_1)^2}{2 \cos^2 \alpha} - \phi \left[\gamma \, (d_2)^2 - \frac{\gamma \, (d_1)^2}{\cos^2 \alpha} \right] \tan \alpha = \frac{\gamma \, (V_1)^2 \, d_1 \cos \alpha}{g \, d_2} \left[d_2 - \frac{d_1}{\cos \alpha} \right] \quad \text{(11)}$$

Eq. 11 is a complete equation for the hydraulic jump in sloping rectangular channels.

For practical use it is desirable to have an equation for the depth of tailwater, d_2. Eq. 11 can be simplified by dividing both sides of the expression by $\frac{\gamma}{2} \left[d_2 - \frac{d_1}{\cos \alpha} \right]$, giving

$$\left(d_2 + \frac{d_1}{\cos \alpha} \right) - 2 \, \phi \left(d_2 + \frac{d_1}{\cos \alpha} \right) \tan \alpha = \frac{2 \, (V_1)^2 \, d_1 \cos \alpha}{g \, d_2} \quad \text{(12)}$$

and Eq. 12 can be solved for d_2:

$$d_2 = \frac{d_1}{2 \cos \alpha} \left[\sqrt{\frac{8 \, (V_1)^2 \cos^3 \alpha}{g \, d_1 \, (1 - 2 \, \phi \tan \alpha)} + 1} - 1 \right] \quad \text{(13)}$$

Letting the quantity $\frac{(V_1)^2}{g \, d_1} = \lambda =$ the kinetic flow factor, Eq. 13 can be written in another convenient form:

$$d_2 = \frac{d_1}{2 \cos \alpha} \left(\sqrt{\frac{8 \, \lambda \cos^3 \alpha}{1 - 2 \, \phi \tan \alpha} + 1} - 1 \right) \quad \text{(14)}$$

Eq. 14 is a general equation for d_2, the depth of flow at section 2, for either Case 3 or Case 4 of the hydraulic jump in sloping channels. For Case 3, the depth at section 2 is equivalent to the tailwater depth.

Case 2, shown in Fig. 1, is a transition between cases 1 and 3, and neither Eq. 14 nor Eq. 3 for the level-floor jump can be applied directly to that case. As shown subsequently, however, experimental coefficients can be used to relate Eqs. 14 and 3 in a solution for Case 2.

Laboratory Investigation of Hydraulic Jump in Sloping Channels

Scope of Tests.—An intensive study of the hydraulic jump in sloping channels was begun in August, 1936, by Mr. Yarnell, in cooperation with the Iowa Institute of Hydraulic Research, State University of Iowa, Iowa City, Iowa. Six hundred tests, on slopes of 1 on 6, 1 on 3, 1 on 2, and 1 on 1, were completed, and an analysis of the data had been started before Mr. Yarnell's death in March, 1937. The data from this investigation were loaned to the TVA in April, 1939.

Unfortunately, because of limitations in the apparatus, jumps tested on slopes greater than 1 on 6 were largely in the Case 2 classification shown in Fig. 1. Since it is basic to the present analysis to derive experimental coefficients from jumps in the Case 3 or Case 4 classification, data on the steeper slopes were insufficient to develop the necessary empirical relations.

Laboratory Setup.—The laboratory setup consisted of a glass-walled flume, 30 in. wide, 3 ft deep, and 30 ft long, with a large head-tank at the upper end. The sloping floor was built of wood, carefully treated and reinforced to produce a smooth, flat surface. The upstream end of the slope was fastened to a watertight bulkhead in the head-tank. An adjustable, sharp-edged, vertical slide-gate was used to regulate flow from the head-tank to the flume. Water depths in the flume were regulated at the lower end by a steel gate hinged at the bottom. Discharges were measured by means of a sharp-edged, suppressed, rectangular weir which emptied into the head-tank. Water-surface elevations in the flume were measured with a point gage on a sliding carriage. Point-gage readings were "zeroed" by readings on the floor after every observation. Three rows of piezometers were located on 6-in. centers over the entire length of the flume. Piezometers in the level steel floor consisted of $\frac{3}{8}$-in. pipe nipples, threaded into the floor and ground flush with the bottom of the flume. Piezometers in the wooden apron were made of $\frac{1}{4}$-in. brass tubing. Each piezometer was connected by means of rubber tubing to a manometer board.

Verification of Analysis from Tests on 1 on 6 Slope

The Kinetic Flow Factor, λ.—In the foregoing analysis, the quantity $\frac{(V_1)^2}{g\,d_1} = \lambda$ has been referred to as the kinetic flow factor, a dimensionless criterion for dynamic similarity which is the square of Froude's number.[8] In problems of open-channel flow Froude's number is applicable when forces of inertia and gravity predominate. As applied to the hydraulic jump in channels of unit width, dynamic similarity implies geometric similarity of all common vertical and longitudinal dimensions. Since the kinetic flow factor is a dimensionless index of similarity, independent of the units of measurement or absolute size of the phenomena, it can be used to correlate experimental data obtained under many different test conditions.

The Pressure Coefficient, ϕ.—In the derivation of Eq. 11 the horizontal component of pressures on the sloping floor was expressed in terms of an experimental coefficient, ϕ (see Eq. 10). In this investigation, normal pres-

[8] "The Analytical Approach to Experimental Hydraulics," by Hunter Rouse, *Civil Engineering*, November, 1934, p. 563.

sures on the sloping floor were measured by means of piezometers. An interesting fact observed by Messrs. Bakhmeteff and Matzke[3] and verified by the Yarnell tests was that the pressure profile of the hydraulic jump on a slope is very closely approximated by the point-gage profile. It would seem possible, therefore, to derive reasonably accurate pressure coefficients without the use of piezometric observations.

If p_n is the normal pressure at any point on the slope, then

$$p_n = \gamma\, y \qquad (15)$$

in which y is the vertical height of the pressure gradient above the floor. Thus, if $\bar{y}$ is the average height of the pressure gradient over a slope of length $\frac{L_r}{\cos \alpha}$, the total normal pressure on a unit width of channel is

$$P_n = \frac{\gamma\, \bar{y}\, L_r}{\cos \alpha} \qquad (16)$$

The quantity ($\bar{y}\, L_r$) in Eq. 16 is the area of the pressure diagram, shown by experiment to be proportional to the area of the jump body. The horizontal component of the total normal pressure is P_s, or

$$P_s = P_n \sin \alpha = \gamma\, \bar{y}\, L_r \tan \alpha \qquad (17)$$

Thus, from Eqs. 10 and 17,

$$\phi = \frac{\bar{y}\, L_r}{(d_2)^2 - \dfrac{(d_1)^2}{\cos^2 \alpha}} \qquad (18)$$

Fig. 6(*a*) shows a plot of experimental data from tests on a 1 on 6 slope, indicating that ϕ is a function of the kinetic flow factor, or

$$\phi = f_1(\lambda) \qquad (19a)$$

An average curve is quite well defined for values of λ from 5 to 50. Future tests may extend the curve to higher values of the kinetic flow factor, unless a maximum of $\lambda = 50$ can be accepted as a practical limit. The curve in Fig. 6(*a*) has the straight-line equation,

$$\phi = 2.58 - 0.021\, \lambda \qquad (19b)$$

Length of the Roller, L_r.—It is convenient to express the length of the jump roller in terms of a dimensionless ratio. In treatments of the level-floor hydraulic jump, the length is commonly expressed in the ratio $\frac{L}{d_j}$, in which d_j is the difference in water-surface elevation at sections 1 and 2. For the case of the jump on a slope, however, d_j is not a suitable parameter since, as shown in Fig. 1, it varies with the jump's position on the slope. Experiments indicate that the length of the roller of the hydraulic jump in sloping channels is best defined by the ratio $\frac{L_r}{d_2}$ as a function of the kinetic flow factor, or

$$\frac{L_r}{d_2} = f_2(\lambda) \qquad (20)$$

Fig. 6(*b*) is a plot of Eq. 20 from experiments on the 1 on 6 sloping channel. An average curve is reasonably well defined.

Verification of Eq. 14, Cases 3 and 4.—To confirm the applicability of Eq. 14, all tests for which the nominal slope of the channel was 1 on 6 were classified

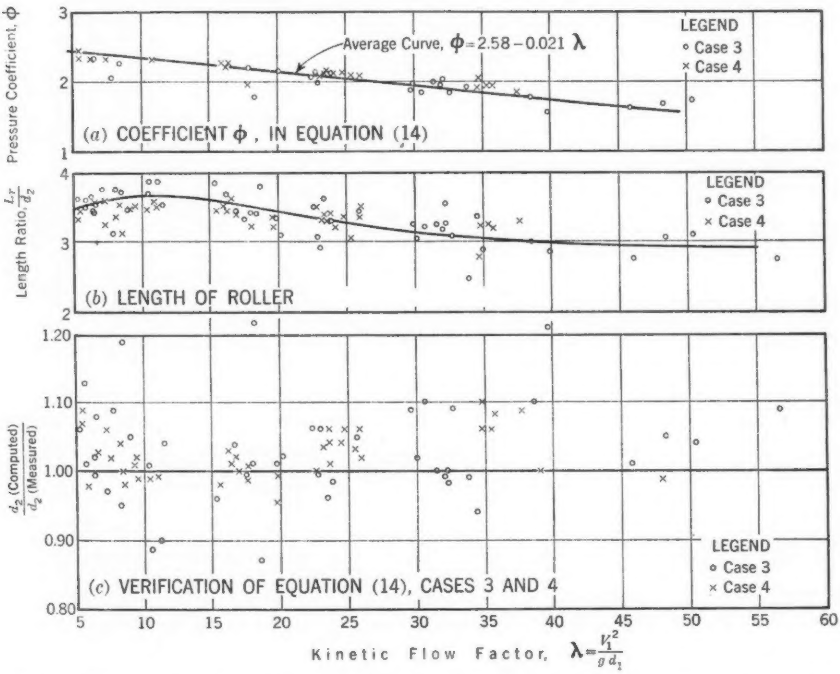


FIG. 6.—DIMENSIONLESS RELATIONSHIPS DERIVED FROM EXPERIMENT (SLOPE OF CHANNEL, 1 ON 6). Eq. 14, Cases 3 and 4

as shown in Fig. 1. For each test classified in Case 3 or 4, the depth d_2 was computed from Eq. 14, using measured values of slope, discharge, and initial depth, with values of ϕ taken from Fig. 6(*a*). Actual slope of the channel varied from 0.170 to 0.176 for different installations. The depth at section 1, d_1, was taken as the average of a series of point-gage measurements made at the toe of the roller. Since the depth d_1 is not only difficult to measure accurately but is important to the evaluation of Eq. 14, it is frequently a source of inaccuracy in the computations. The depth at section 2, d_2, was taken for Case 3 jumps as the depth of tailwater and for Case 4 jumps as the depth at the end of the roller.

A comparison for verification of Eq. 14 is shown in Fig. 6(*c*), in which the ratio of d_2-computed to d_2-measured is plotted against the kinetic flow factor. It is believed that deviations from an exact agreement between measured and computed values reflect to some extent the difficulties of observation.

Verification of Eq. 14, Case 2.—Fig. 7 shows Case 2 of the hydraulic jump which, being partly on the slope and partly on the level floor, is intermediate

between cases 1 and 3. Obviously, if given values of slope, discharge, and initial depth for a Case 2 jump are evaluated in Eq. 14, the result, always exceeding the depth of tailwater, d_2, for the given case, is a depth $(d_2)'$ for an imaginary Case 3 jump. Similarly, if the same data are evaluated in Eq. 3, the result is a depth $(d_2)''$, less than d_2, for an imaginary Case 1 jump.

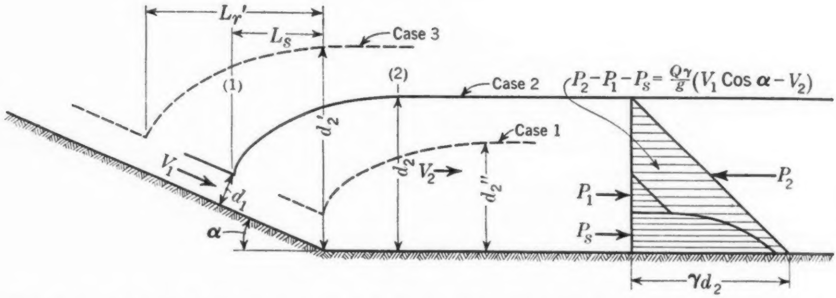


FIG. 7.—CASE 2; THE HYDRAULIC JUMP IN SLOPING CHANNELS, WITH THE TOE OF THE ROLLER ON THE SLOPE AND THE END OF THE ROLLER ON THE HORIZONTAL FLOOR

Experiments indicate that the depth of tailwater for any jump in the Case 2 classification can be expressed in an empirical relation,

$$d_2 = (d_2)' - k\left[(d_2)' - (d_2)''\right] \qquad (21)$$

in which k is an experimental coefficient, a function of λ and the ratio $\frac{L_s}{(L_r)'}$. The symbol L_s denotes the horizontal length from the toe of the jump to the lower end of the slope, and $(L_r)'$ is the length of the roller of the imaginary Case 3 jump from Eq. 20. Thus,

$$k = f_3\left[\frac{L_s}{(L_r)'}\right] \qquad (22)$$

Fig. 8 shows a plot of k against $\frac{L_s}{(L_r)'}$ from tests on the 1 on 6 slope. For all practical purposes, the average curve ($\lambda = 25$) is probably sufficient. Fig. 9 for Case 2 jumps in the 1 on 6 sloping channel shows a comparison of measured depths with depths computed from Eqs. 3, 14, and 21, using the average curve of Fig. 8.

Practical Application of the Analysis.—Frequently, when a large number of computations are required, solution for d_2 in Eq. 3, Case 1, can be simplified by writing the equation in dimensionless form,

$$\frac{d_2}{d_1} = \tfrac{1}{2}\left(\sqrt{8\,\lambda + 1} - 1\right) \qquad (23)$$

and the ratio $\frac{d_2}{d_1}$ can be plotted as a function of λ.

Similarly, for cases 3 and 4, when the slope of the channel is constant, Eq.

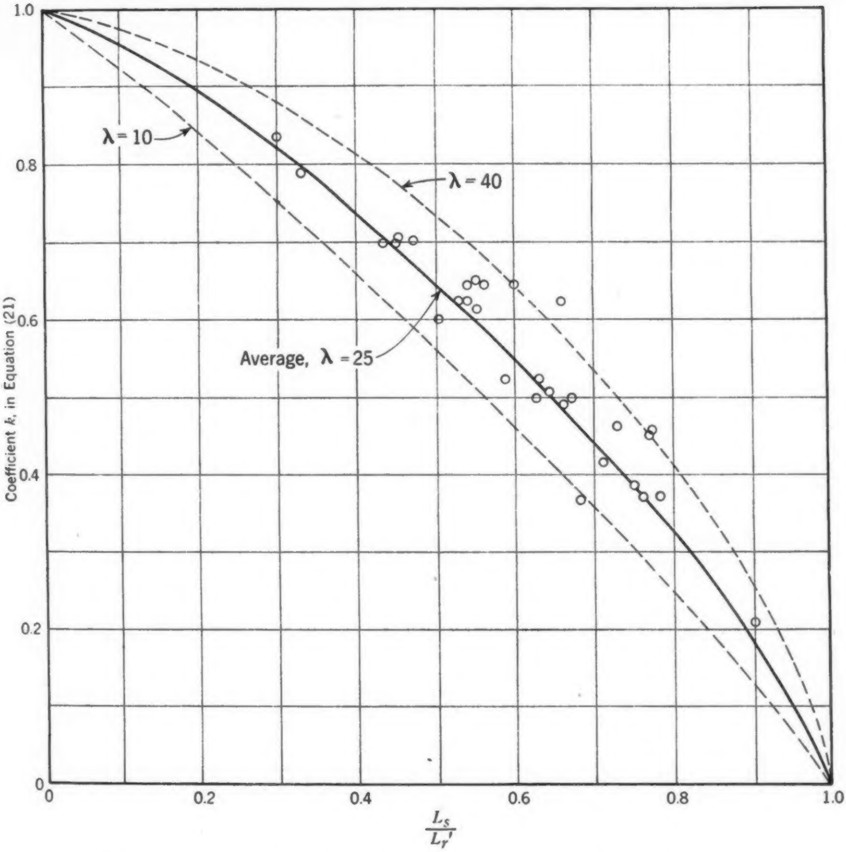


FIG. 8.—COEFFICIENT k IN EQ. 21, FOR A CASE 2 JUMP ON A 1 ON 6 SLOPE

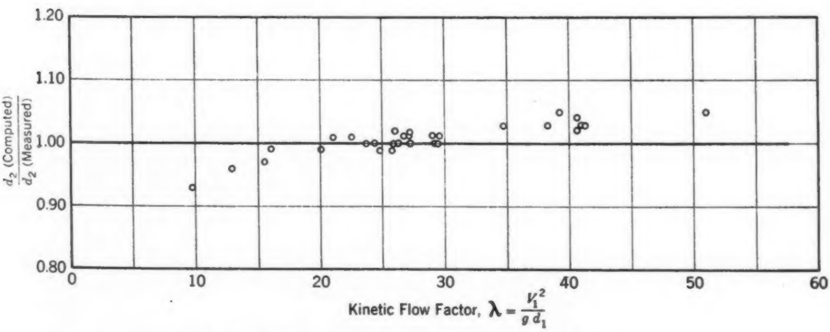


FIG. 9.—VERIFICATION OF EQ. 21, CASE 2, 1 ON 6 SLOPE

14 can be written,

$$\frac{d_2}{d_1} = \frac{1}{2 \cos \alpha} \left(\sqrt{\frac{8 \lambda \cos^3 \alpha}{1 - 2 \phi \tan \alpha} + 1} - 1 \right) \ldots\ldots\ldots\ldots (24)$$

and again, if the terms $\cos \alpha$ and $\tan \alpha$ are evaluated, the ratio $\frac{d_2}{d_1}$ can be plotted as a ratio of λ, using ϕ from Eq. 19*a*. Having curves of Eqs. 23 and 24 for cases 1 and 3, the solution of Eq. 21 for Case 2 is greatly facilitated.

In nearly every practical case, the location of section 1 at the toe of the jump must be assumed in first approximation for d_2, in order that d_1, λ, and ϕ can be evaluated from known conditions. If the jump falls in the Case 2 classification, the length L_s must also be assumed. Results obtained from the initial assumptions can readily be refined by successive approximations.

Conclusions

It is convenient to classify the most common forms of the hydraulic jump in open channels into several general cases based on the position of the jump in a sloping channel. Although the jump on a slope is similar in most respects to the level-floor jump, analysis of this case is dependent upon experimental coefficients to evaluate pressures on the sloping floor.

This paper presents a generalized analysis of the jump in sloping channels, leading to a practical method of computing the essential dimensions of the jump. The treatment was verified by application to experimental data on jumps in a 1 on 6 sloping channel. Dimensionless presentation of data on the 1 on 6 slope makes the results suitable for design use. It is hoped that further experimentation will make available similar data for use with steeper slopes.

Acknowledgments

The analysis contained in this paper was made in the Hydraulic Laboratory Section of the Hydraulic Data Division, Tennessee Valley Authority. The work of the Hydraulic Laboratory is under the direction of A. S. Fry, M. Am. Soc. C. E., head hydraulic research engineer, and the direct supervision of G. H. Hickox, M. Am. Soc. C. E., senior hydraulic engineer.

As stated previously, the experimental data were obtained from the laboratory investigation made in 1936–1937 by Mr. Yarnell, whose notes and photographs were loaned to the TVA by the U. S. Department of Agriculture. The writer was employed as an assistant to Mr. Yarnell from August, 1936, until January, 1937.

Grateful acknowledgment is due Mr. Hickox and Sherman M. Woodward, M. Am. Soc. C. E., for consultation and advice. Many others have contributed with criticism and assistance in computations.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

APPLICATION OF SOIL MECHANICS IN DESIGNING BUILDING FOUNDATIONS

By A. Casagrande,[1] Assoc. M. Am. Soc. C. E., and R. E. Fadum,[2] Jun. Am. Soc. C. E.

Synopsis

The first part of this paper reviews the causes of building settlements, and summarizes what soil mechanics has to offer as a basis for the design of building foundations. In the second part are described the foundations of two large office buildings in Boston, Mass., which were designed specifically for the purpose of reducing total and differential settlements to a tolerable amount. The settlements of these buildings are compared with those of other large buildings designed without the use of soil mechanics. The methods of reducing existing differential settlements are not discussed. The success of any method such as underpinning, the application of additional load to the lighter sections of a structure, the provision of continuous jacking arrangements, or the method of "bleeding" the underground by means of borings adjacent to and under those parts of a structure that have not settled are dependent on accurate observations of the behavior of the structure which is to be remedied.

Introduction

Although problems dealing with building settlements may be relatively simple as compared with other problems in soil mechanics, knowledge of soi properties is not yet sufficiently developed to permit an accurate solution even of this problem. It must be kept in mind that, in general, even the most elaborate underground explorations do not disclose a complete picture of the soil conditions. Therefore, the results of a settlement analysis, as well as of most problems in applied soil mechanics, should be expressed by a range which includes the possible limits of all factors entering the computations. A clear

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **April 1, 1943**.

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discussion of the possible variations of each factor should be presented to facilitate a final decision by those who use the information for design purposes. It cannot be emphasized too strongly that the correct application of the results of such an investigation to the design of earthworks or foundations will, in addition to a thorough knowledge of soil mechanics, always require experience and judgment.

Differences Between Soils and Structural Materials.—The deformation of soils, even under small loads, is much larger than that of steel or concrete. Furthermore, the deformation of many types of soils does not occur simultaneously with the application of a load; it develops in the course of time. Therefore, one may not be aware of a dangerous situation until a building begins to crack many years after it has been built.

Another difference between structural materials and soils resides in the fact that in the former deformation is chiefly a result of a change in shape. In soils, however, deformation is the result of both a change in shape and a change in volume. In some problems, and in particular problems dealing with the settlement of buildings resting on clay, the deformation due to change in volume of the underlying soil layers is usually much more important than the deformation due to change in shape.

Much confusion and harm have resulted in the past from the belief that a satisfactory allowable load for a soil can be obtained by reducing the failure load, derived from surface-loading tests or pile-loading tests, by a liberal factor of safety. The resulting stresses will not produce rupture of the soil. There is no assurance, however, that these "safe" stresses in the soil will not produce objectionable or dangerous settlements.

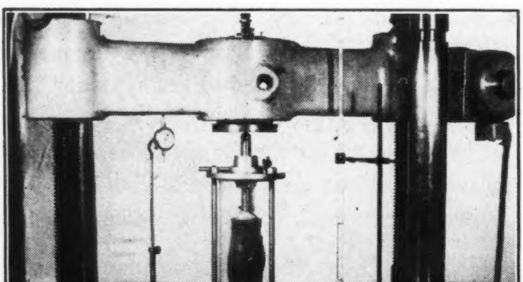


FIG. 1.—TRIAXIAL COMPRESSION APPARATUS

SOIL PROPERTIES RESPONSIBLE FOR SETTLEMENTS

General. — Since, for practical purposes, soils possess no tensile strength, only the deformation characteristics resulting from compression forces are of interest. Therefore, the principal testing methods are designed to subject soil samples to compression. Theoretically, it should be possible to determine all the stress-deformation characteristics from so-called "triaxial compression tests." In this type of test the principal stresses can be varied at liberty, thereby producing any desired combination of normal and shearing stresses within a sample. Actually, the only practical type of triaxial compression test apparatus thus far developed is one in which two of the principal stresses

are kept equal. These stresses are produced by the pressure of a liquid surrounding a cylindrical specimen (Fig. 1). If such a compression test is made without this liquid pressure, the test is called an "unconfined compression test." This type of test is similar to cylindrical compression tests used in concrete testing. In a third type of compression test, which is of particular importance in the testing of clays, the specimen is confined laterally in a metal ring and is compressed between two porous plates. In this "confined compression test" (which is also called a "consolidation test"), the lateral displacements are prevented entirely, and one measures only the relation between stress, volume, and time.

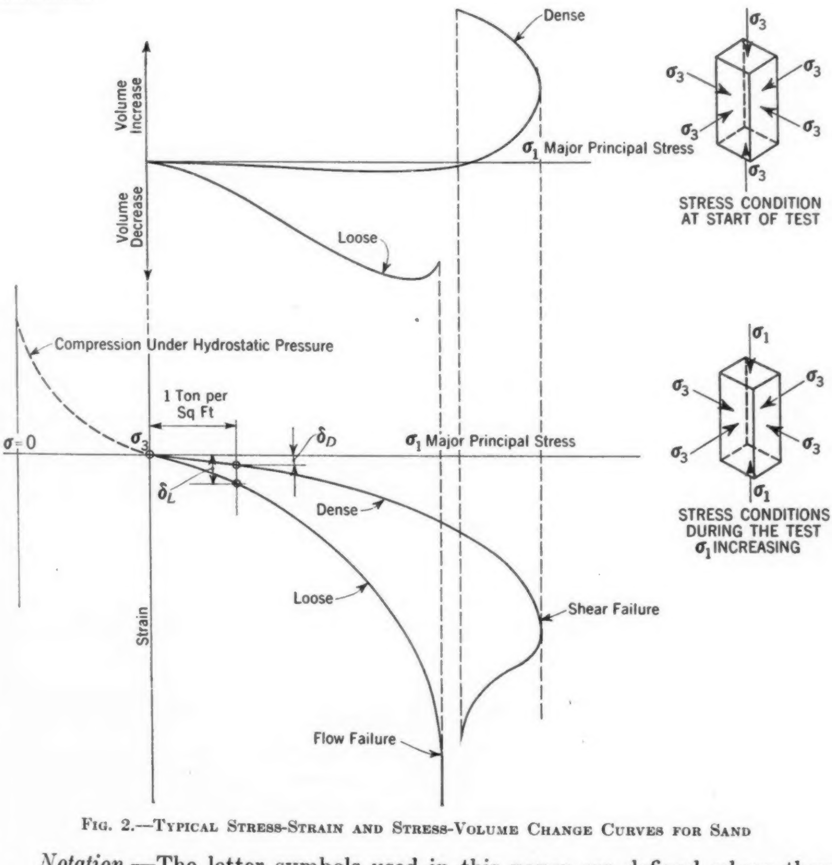


FIG. 2.—TYPICAL STRESS-STRAIN AND STRESS-VOLUME CHANGE CURVES FOR SAND

Notation.—The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference in the Appendix.

Stress-Deformation Characteristics of Cohesionless Soils.—When a sample of sand is tested in triaxial compression, starting, for example, from a hydrostatic stress condition, and then one of the principal stresses is increased until failure occurs, stress-strain and stress-volume change curves of the type shown in Fig. 2 are obtained. A loose sand decreases in volume, whereas a dense sand

expands during loading. Although this behavior is of great importance in connection with the question of stability of dams consisting chiefly of fine-grained or medium-grained sands, it is of limited interest in problems attending building construction. Buildings resting on loose fine sands or inorganic silts are likely to undergo sudden settlements if the underlying soil is disturbed—for example, by heavy pile-driving operations in adjacent building areas. Most sand deposits, however, are sufficiently dense that no such dangers exist.

The chief concern is with ordinary deformation under stress and its effect upon the distribution of soil reactions. It can be noted from Fig. 2 that an increase in stress of (for example) 1.0 ton per sq ft produces a larger strain in the loose sand than in the dense sand. If the two tests are repeated on specimens of the same material, subjected to different intensities of minor principal stress, one finds that the strain corresponding to $\sigma_1 - \sigma_3 = 1.0$ ton per sq ft changes approximately in inverse proportion to the minor principal stress σ_3. Furthermore, it is found that the shear strength of sands increases approximately in direct proportion with the minor principal stress. The coefficient expressing this proportionality is called the angle of internal friction.

The fact that the resistance to deformation, as well as the ultimate strength, of a cohesionless solid increases approximately in direct proportion to the confining pressures can be demonstrated easily by placing sand in a rubber bag. When the air is evacuated from the interior of such a soft bag, the contents become as hard as a brick. The evacuation of air subjects the sand to a pressure corresponding in intensity to that produced by approximately 20 ft of soil overburden.

From consideration of these physical characteristics of sands, and from observations, the following conclusions, applicable only to noncohesive soils that extend to a depth below the loaded area for a distance at least equal to the width of the loaded area, may be drawn:

(*a*) The intensity of pressure exerted on a rigid, loaded area supported on the surface of a cohesionless soil varies in such a manner that a maximum occurs at the center and no pressure occurs at the edge. For small footings for which the product of the width of the footing and the unit weight of soil ($b\,\gamma$) is small in relation to the average unit load on the footing $\left(p = \frac{P}{A}\right)$, the pressure diagram is parabolic in shape, as shown in Fig. 3(*a*). When $b\,\gamma$ is of the same order of magnitude as $p = P/A$ (which is usually the case for large footings, or for a rigid foundation mat), the diagram of soil reactions is as shown in Fig. 3(*b*).

(*b*) As the depth of a rigid, loaded area beneath adjacent ground surface is increased, the soil reactions change until the intensity of pressure is nearly uniform, as shown in Fig. 3(*c*). This is frequently the case in caissons and pier foundations.

(*c*) The ultimate bearing capacity of a cohesionless soil increases approximately in direct proportion to the width of a loaded area.

(*d*) The ultimate bearing capacity of a cohesionless soil increases approximately as the square of the depth of the loaded area beneath adjacent ground surface.

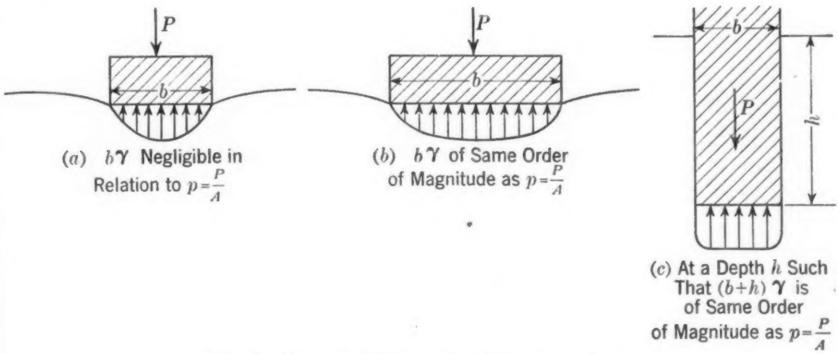


FIG. 3.—RIGID FOOTING ON THE SURFACE OF SAND

(*e*) Specifications for the allowable loads for sands, as generally given in building codes, do not take sufficiently into consideration the favorable influence of depth. Such "bearing values" are far too conservative for the case of footings founded at considerable depth, whereas, for narrow, interior, wall footings that are founded at the surface or at a shallow depth beneath immediately adjacent ground surface, the values are not sufficiently conservative.

(*f*) When fairly heavy loads are supported at shallow depths on narrow footings resting on loose sand, it is well to test the bearing capacity by loading a test area of similar width. However, for large areas on dense sand, or for areas which are to be founded at a considerable depth below ground surface on dense sand, even the heaviest loading which building codes permit is exceedingly conservative, so far as strength of the soil is concerned, and requires no verification by load tests.

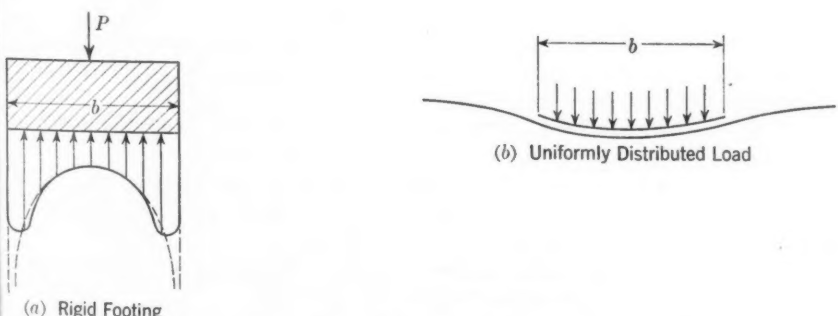


FIG. 4.—LOADS ON THE SURFACE OF ELASTIC MATERIAL

(*g*) From the theory of elasticity it follows that, for a perfectly elastic soil, the reactions on rigid areas and the settlements of uniformly loaded areas on the ground surface should be as shown in Fig. 4. The variations in pressure

intensities and the settlement patterns thus obtained are just the opposite from those obtained in corresponding cases for sand, as shown in Figs. 3(*a*) and 5(*a*), respectively. Thus, in the case of sand, less harm would result from

(*a*) $b\gamma$ Negligible in Relation to p

(*b*) $b\gamma$ of Same Order of Magnitude as p

FIG. 5.—UNIFORMLY DISTRIBUTED LOAD ON THE SURFACE OF SAND

just guessing intelligently at the deflections or the distribution of soil reactions than from making elaborate computations on the basis of hypothetical theories which definitely do not apply. Furthermore, the fact that Hooke's law does not apply to cohesionless soils also invalidates the law of superposition. (Modifications of Boussinesq's formula,

$$\sigma_z = \frac{3\,P}{2\,\pi}\frac{z^3}{R^5} \qquad (1a)$$

by introducing a "concentration factor," α, in the form,

$$\sigma_z = \frac{\alpha\,P}{2\,\pi}\frac{z^\alpha}{R^{\alpha+2}} \qquad (1b)$$

yield, when integrated over finite areas, essentially the same type of distribution as shown in Fig. 4. Therefore, this hypothetical solution, utilizing a concentration factor, is no improvement over the theory of elasticity. Both are equally unsuited for computing soil reactions and deflections on sand.)

(*h*) The settlement of a uniformly loaded area is a minimum in the center and a maximum along the edge. The deflection curve is as shown either in Fig. 5(*a*) or 5(*b*), depending on the relative magnitude of $b\,\gamma$ and p.

(*i*) The average settlement corresponding to a given unit load increases for small areas with increasing width of the footing, whereas for large areas it is practically independent of the width.

(*j*) The settlement for a given unit load decreases rapidly as the depth of the loaded area beneath adjacent ground surface increases.

(*k*) Sand layers provide an excellent medium for spreading load and for reducing differential settlements, due to the small strain which corresponds to ordinary loads and to the fact that the resistance of sand to deformation increases in proportion to the applied pressure. The presence of sand strata should be utilized as much as possible for this purpose.

(*l*) Conventional design of building foundations by means of individual column footings and wall footings, with or without piles, is in general entirely adequate for foundations on medium-grained and coarse-grained cohesionless soils, provided that borings to a depth at least equal to the width of the building have verified the fact that the sand is not underlain by a compressible stratum.

(*m*) When founding on very fine loose sand, settlements may become excessive. Very fine loose sands, rock flour, etc., may settle seriously at some later time, due to a lowering of the ground-water table, the driving of piles, or other disturbances caused by near-by construction operations.

Stress-Deformation Characteristics of Clays and Other Cohesive Soils.—Clays are very fine-grained soils having grain sizes which run well into the colloidal range. Such soils, when dried, become hard and appear like a soft rock. When remolded at consistencies between certain limiting water contents, clays exhibit the property of plasticity. With relatively little experience, clays can be identified in the field from an examination of these properties.

Since the plasticity, as well as the compressibility and permeability of a clay, is related to its content of colloidal, scale-like particles, the Atterberg liquid and plastic limit tests are useful for general classification purposes. Such classification tests, however, are far from adequate in quantitatively describing the pertinent properties of an undisturbed clay. With the same raw material, nature has made clays possessing widely different structural characteristics. These are dependent on many factors, as, for example, whether sedimentation took place in fresh or salt water and the amount of load under which consolidation proceeded.

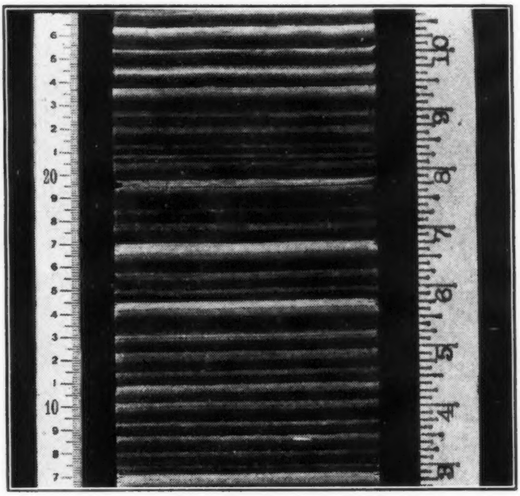


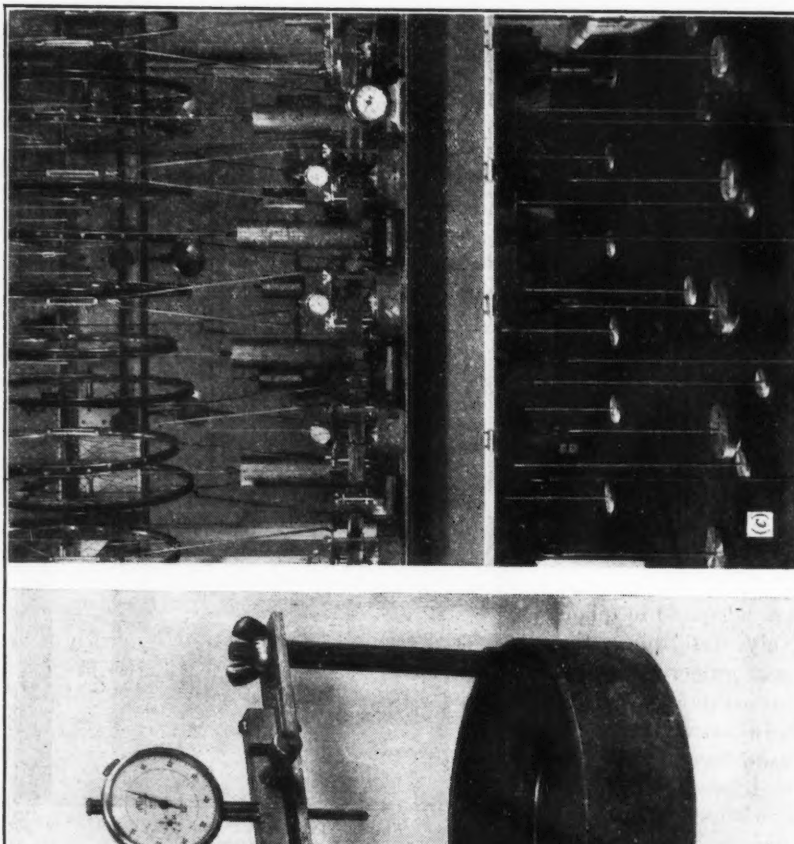
FIG. 6.—TYPICAL VARVED CLAY

Many undisturbed clays possess great strength in relation to their high void ratios.[3] A piece of such a clay can be broken like a soft solid, showing distinct rupture surfaces similar to those of brittle materials. However, if remolded without the addition of water, it becomes a plastic and very soft mass bearing no resemblance to the undisturbed soil.

Most undisturbed clays behave like a perfectly incompressible material immediately upon application of a load. If a load is maintained, however, water gradually will be squeezed out of those parts of the clay which are stressed, until the mineral skeleton is compressed to a volume at which it can carry the load. This process, which is called "consolidation," can be analyzed by means of the theory of consolidation[4] set forth by Karl Terzaghi, M. Am. Soc. C. E.

[3] "The Structure of Clay and Its Importance in Foundation Engineering," by A. Casagrande, *Journal*, Boston Soc. of Civ. Engrs., April, 1932.

[4] "Erdbaumechanik auf bodenphysikalischer Grundlage," by Karl Terzaghi, Leipzig, Deuticke, 1925. For up-to-date presentation, see "Theorie der Setzung von Tonschichten," by Karl Terzaghi and O. K. Fröhlich, Vienna, 1936.



(a) and (b) Detail Views

(c) Setup in a Loading Frame

FIG. 7.—CONSOLIDATION APPARATUS

Deformation of Clay Without Volume Change.—Stress-strain curves obtained from unconfined compression tests or surface-loading tests, on most undisturbed clays, are almost straight lines within the load range of working stresses. Hence, it should be possible to determine that part of the settlement of a structure which occurs during the application of load, and which is due to lateral deformation without volume change, by simply determining the modulus of elasticity and then applying appropriate formulas from the theory of elasticity. This approach has been tried by many investigators, and almost invariably it has been found that the actual settlements are only a fraction of the computed settlements. Some of this discrepancy is frequently due to the fact that the so-called "undisturbed samples" have suffered some disturbance during the sampling operations. In many cases this discrepancy is due to anisotropic characteristics of natural clay deposits.[5] Most clays are stratified, even though the stratification may not be visible. The most pronounced stratification is found in the so-called "varved" clays, in which thin layers of rock flour are closely laminated with clay layers of varying degrees of plasticity and frequently of varying colors. An excellent example of such a clay is shown in Fig. 6.

Whenever any type of stratification is present, the soft layers are confined laterally, to a certain extent, by the stronger sand and silt layers or partings. Hence, under the load of a building, that part of the settlement which is due to the lateral deformation of the underlying clay strata can develop only to a limited extent, as compared with the settlement which would take place if the clay were homogeneous and isotropic.

Consolidation of Clays.—There are many types of consolidation apparatus in use, all of which incorporate the principle, introduced by Professor Terzaghi, of compressing a thin cylindrical sample confined in a ring between two porous stones. The consolidation apparatus shown in detail in Fig. 7(*a*) and set up in a dead-weight loading frame in Fig. 7(*c*) is a simple and efficient example.[6] The test is performed by applying the load in increments. Each increment is allowed to remain on the sample a sufficient length of time for consolidation to take place, the extensometer being read at appropriate time intervals. The data of such a test are plotted in the form of a "pressure-void ratio curve" (Fig. 8) and a series of "consolidation curves" (Fig. 9).

Without presenting Professor Terzaghi's theory of consolidation of fine-grained soils in detail, the following points will be mentioned: The time required to reach a given degree of consolidation is directly proportional to the square of the height of a layer of clay, drained on both sides; it is inversely proportional to the coefficient of permeability; and it is directly proportional to the slope of the pressure-void ratio curve.

It is a simple matter to convert consolidation curves obtained in the laboratory to a curve showing the probable progress of settlement with time for a structure supported on the material from which the samples were obtained,

[5] "Influence of Anisotropic Properties of Clay Deposits on Settlements," by Austin B. Mason, a thesis presented to the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in June, 1939, in partial fulfilment of the requirements for the degree of Doctor of Science in Engineering.

[6] "Notes on Soil Testing for Engineering Purposes," by A. Casagrande and R. E. Fadum, *Soil Mechanics Series No. 8*, Graduate School of Eng., Harvard Univ., Cambridge, Mass., January, 1940, p. 42.

provided the thickness and drainage characteristics of the loaded clay layers are known accurately. Unfortunately, this is seldom the case. When the soil conditions are complex, a tolerably accurate knowledge of the drainage characteristics can be obtained only by means of detailed underground explorations in which continuous samples are obtained. Even such an extensive study will provide information sufficient only to establish certain limits within which the actual conditions may lie. The range thus established may be quite wide.

Preconsolidation Load.—Correct interpretation of the results of consolidation test data is an important prerequisite for a comprehensive settlement analysis. The relation between pressure and void ratio, which is obtained from a consolidation test, may differ widely from the true relationship which a clay

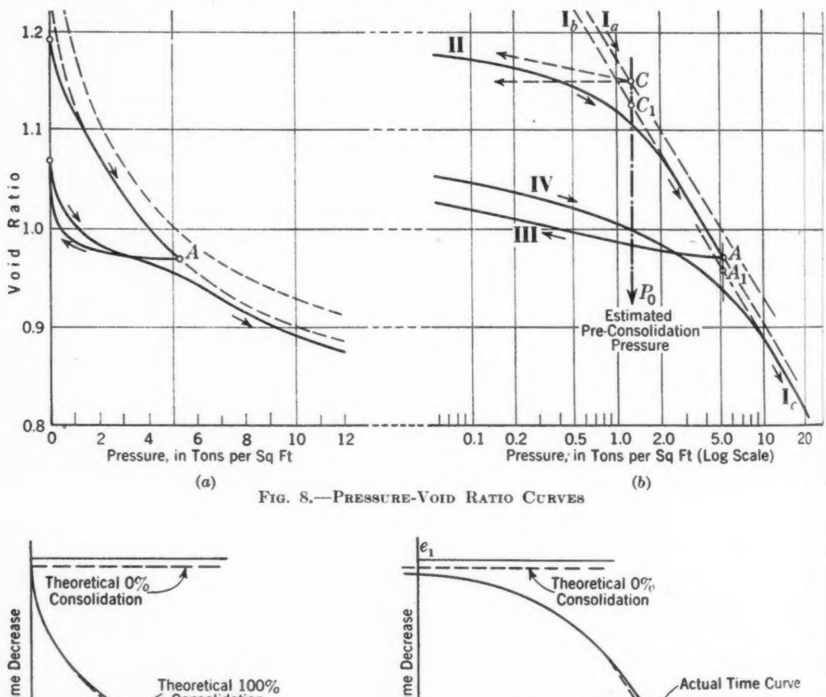


FIG. 8.—PRESSURE-VOID RATIO CURVES

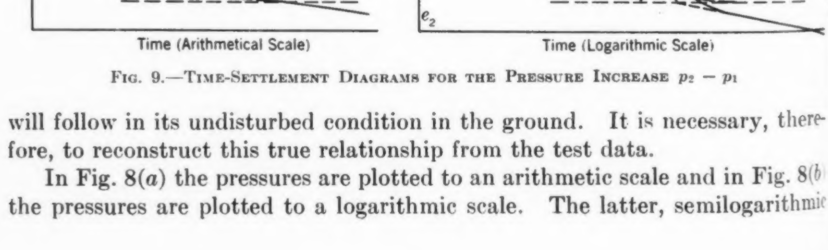


FIG. 9.—TIME-SETTLEMENT DIAGRAMS FOR THE PRESSURE INCREASE $p_2 - p_1$

will follow in its undisturbed condition in the ground. It is necessary, therefore, to reconstruct this true relationship from the test data.

In Fig. 8(*a*) the pressures are plotted to an arithmetic scale and in Fig. 8(*b*) the pressures are plotted to a logarithmic scale. The latter, semilogarithmic

plot lends itself readily to an analysis of the loading history of a sample. The first part (II) of the compression curve is in reality a recompression curve which meets the virgin branch I_b and then continues along that branch as a straight line. At an arbitrary load, corresponding to point A, the load is reduced by a series of decrements to zero, and a rebound curve III is obtained. When load is applied again as before, the recompression curve IV is obtained. This latter curve meets the virgin compression curve I_c at a point corresponding to a higher pressure than that corresponding to point A. This diagram is typical of all very fine-grained soils. The magnitude of the drop in the position of a virgin branch after each rebound cycle depends chiefly on the structural characteristics of a soil.

The similarity in the shape of branch II, the relative positions of branches II and I_b with the shape of the recompression curve IV, and the relative positions of curves IV and I_c suggest that it should be possible to estimate the so-called preconsolidation pressure p_0, under which the soil was consolidated in the ground, from the data of a properly conducted consolidation test.

From a large number of tests on different types of soils, it has been found that the preconsolidation pressure for most clays can be determined with a satisfactory degree of accuracy by means of the empirical method shown in Fig. 10 and explained as follows: First, the position of the virgin compression

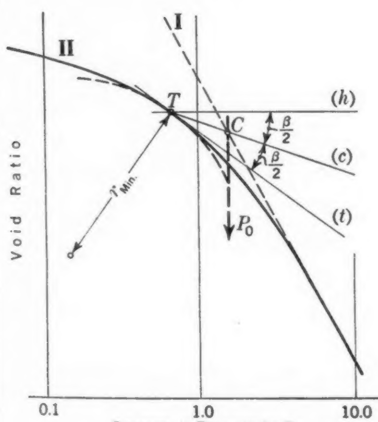


FIG. 10

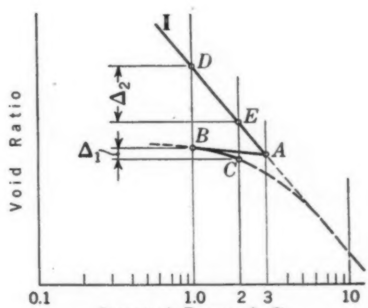


FIG. 11

line (I) is established by extending the straight-line part of the pressure-void ratio curve. Then, on the branch marked II, a tangent (t) and a horizontal line (h) are drawn at the point T which corresponds to the point having the smallest radius of curvature. The angle between (t) and (h) is then bisected, and the abscissa of the point of intersection C, of the bisector (c) with the virgin line I, determines the magnitude of the preconsolidation pressure (P_0).

The significance of this pressure can be demonstrated as follows: The slope of the virgin compression branch on a semilogarithmic plot is not noticeably affected by swelling and minor deformations of a sample. Hence, if a soil is

completely consolidated under a pressure p_1 produced by an existing overburden, the compression due to any additional load Δp is determined easily by multiplying the thickness of the compressible stratum by the ratio of the change in void ratio $(e_1 - e_2)$ corresponding to a change in stress $(p_2 - p_1)$, divided by one plus the initial void ratio $(1 + e_1)$.

If the largest overburden to which a soil has been consolidated during its geologic history later has been partly removed, the compression due to the

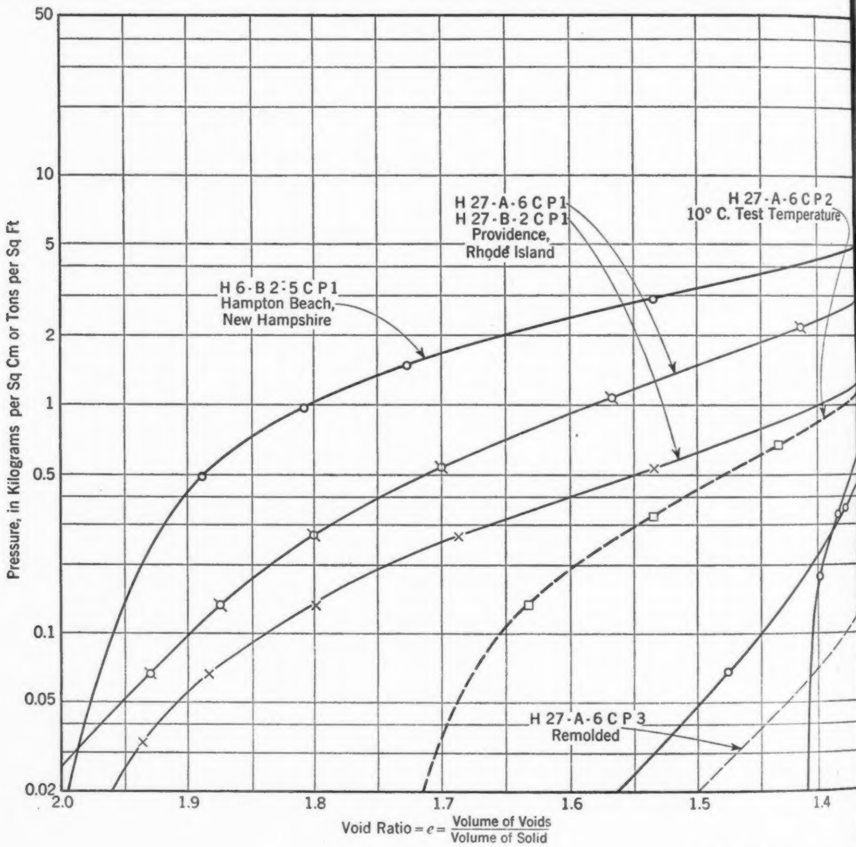
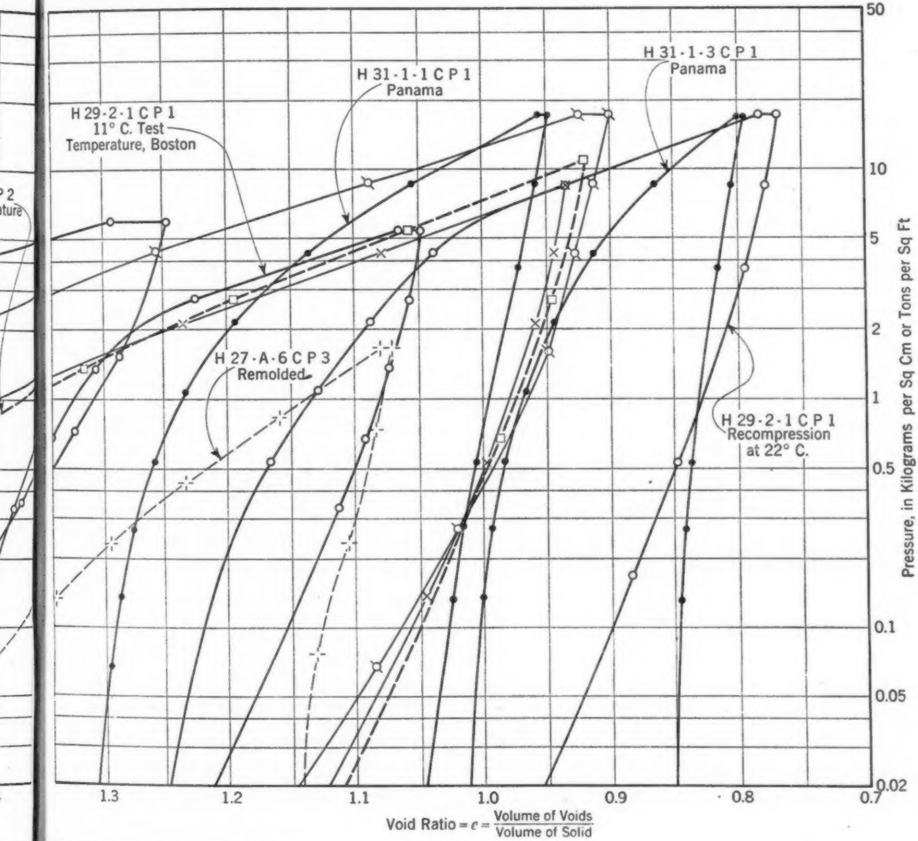


FIG. 12.—COMPRESSION

same pressure increase Δp will be much smaller, in spite of the fact that the virgin compression curve remains unchanged. For example (Fig. 11), if a clay layer has been compressed by an overburden pressure of 3 tons per sq ft and this pressure is later reduced by erosion to 1 ton per sq ft and then again increased to 2 tons per sq ft by the weight of a building, the consolidation due to the building load will correspond to a change, $\Delta e''$, in void ratio. If, however, the clay has never been consolidated by a pressure in excess of the existing overburden pressure of 1 ton per sq ft, the additional stress due to the

weight of the building would have produced a change, $\Delta e'$, in void ratio, which is approximately five times as great as $\Delta e''$. This example should suffice to illustrate the fundamental importance of a careful study of the preconsolidation pressure.

Many attempts have been made to arrive at a reasonable estimate of the compressibility of clays from tests which are more simple than the time-consuming consolidation test and which preferably could be performed on



DIAGRAMS OF ORGANIC CLAYS

small, inexpensive samples. Utilizing a discovery by P. C. Rutledge,[7] Assoc. M. Am. Soc. C. E., that the pressure-void ratio curves for clays of the same general character follow a "pattern," it is possible in many cases to determine the compressibility approximately from the natural water content of the soil. This method will be explained by means of an example.

Example of a Settlement Analysis Based on Simple Soil Tests.—A power

[7] "Compression Characteristics of Clays and Application to Settlement Analysis," by P. C. Rutledge, a thesis presented to the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in June, 1939, in partial fulfilment of the requirements for the degree of Doctor of Science in Engineering.

station had suffered serious settlement in spite of the "fact" that borings showed only sand to a depth of 35 ft and that piles had been driven to a depth of 25 ft to 30 ft. The writers were requested to make a preliminary investigation of the foundation conditions for a proposed extension of the plant. Four exploratory borings were made which revealed a stratum of slightly organic clay extending from approximately 39 ft to 51 ft beneath the ground surface. The clay stratum was underlain by a thick stratum of very compact sand. Almost continuous cores were obtained from the clay stratum by means of 4-ft sections of thin-walled steel tubing, 2 in. in diameter. The entire stratum was found to be fairly homogeneous, rather brittle in the undisturbed condition, and very soft and sticky when remolded. The natural water content ranged between 37% and 58%, the liquid limit between 37% and 65%, and the plastic limit between 20% and 27%. (Whenever the natural water content of a clay, which is consolidated by at least 40 ft of overburden, is almost as high as the liquid limit, it is an indication that the clay has the type of complicated structure which probably results from sedimentation in sea water. Numerous shells which were found in the clay were identified as being of marine origin.)

From the general conditions at the site and the appearance of the clay stratum, it seemed likely that at no previous time did the clay layer carry a heavier overburden than that which existed at the time of inspection. It was evident also that this relatively thin stratum was fully consolidated under the existing overburden. The preconsolidation pressure was computed from the unit weights and thicknesses of the overlying materials at a depth of 45 ft, which corresponded to the mid-depth of the clay stratum, and was found to be approximately 1.5 tons per sq ft. The average water content of the clay stratum was 48%, corresponding to a void ratio of $e = 1.30$. The problem thus was reduced to finding the slope of a virgin compression curve for an organic clay which would compress under a load of 1.5 tons per sq ft to a void ratio of 1.30.

Pressure-void ratio curves for a number of different organic clays and silt-clays are assembled in Fig. 12 (rotated 90° to accommodate the page size). It happens that the curve for the soil used in test No. H27-B-2CPl passes practically through the point corresponding to $p = 1.5$ tons per sq ft and $e = 1.30$. If this were not the case, however, it would be a simple matter to draw a line through this point having the same slope as the other virgin compression curves in the immediate vicinity. Assuming that the average increase in stress due to the contemplated structure is 1.0 ton per sq ft, this increase in stress, according to Fig. 12, will reduce the void ratio of such a clay by about $\Delta e = 0.11$. Therefore, the total estimated compression of the clay stratum under the proposed load will be equal to $\frac{\Delta e}{1 + e}$ times the thickness of the layer—that is, $\frac{0.11}{2.3} \times 12 = 0.57$ ft.

Strength of Clay and Safety Against Rupture.—The question of safety against actual failure of the soil arises in connection with the question of the allowable pressure which a spread footing, pier, or caisson may exert on a

soil. It cannot be overemphasized that a safe load, as far as rupture of the soil itself is concerned, may be anything but safe as far as the structure as a whole is concerned.

In a preceding section it was stated briefly that, with the exception of the case of a heavy load on narrow footings supported on or slightly below the ground surface, the possibility of stressing a cohesionless material to the point of rupture practically does not exist. In the case of clays, however, the situation is quite different.

The simplest approach to the problem of determining the ultimate bearing capacity of a clay is one which was originally suggested by Professor Terzaghi.[4] He assumed a wall footing of width 2 b resting directly on the ground surface, and assumed that the soil was divided into three squares as shown in Fig. 13(a).

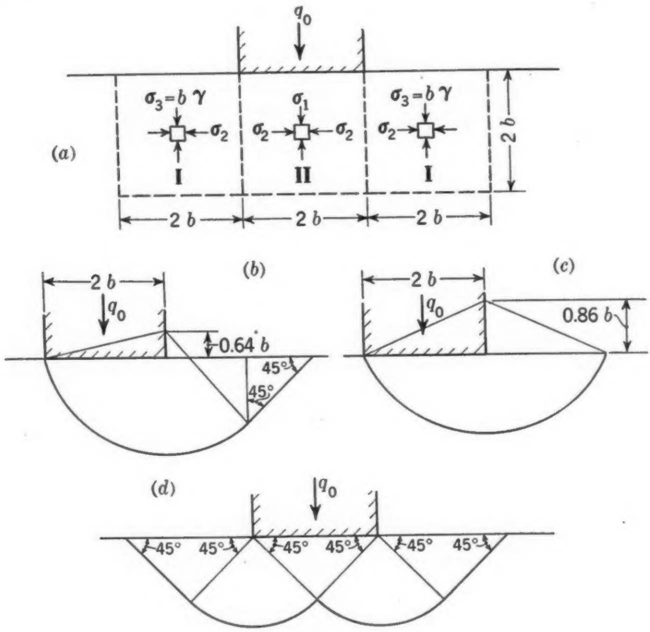


Fig. 13

If the application of a load produces failure of the soil, the underlying prism will deform laterally. The resistance offered to this deformation will be dependent on the resistance to failure of the adjacent prisms.

If a load p per unit of area is applied quickly enough so that a clay will have no time to consolidate, the shear strength will be equal to a constant c; the Mohr rupture curve is represented by a straight line parallel to the pressure axis as shown in Fig. 14(a). Hence, if a clay is subjected to a quick loading, it may be considered to act as a purely cohesive material. This constant shear strength is frequently called "the cohesion of a clay." (The term "cohesion" has been used loosely in soil mechanics and has led to much confusion. In

most cases it is used to designate the shear strength under zero normal pressure. This strength can vary between wide limits for the same clay.)

Referring to Fig. 14(*a*), circle I represents Mohr's circle of stress for the rupture condition of a small element in the center of prism I. The major principal stress, $\sigma_z = b\,\gamma + 2\,c$, which will produce failure, must be equal to

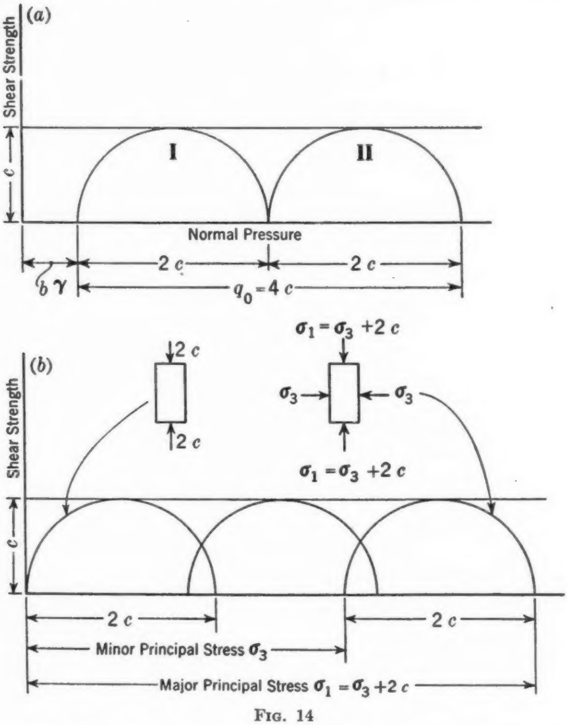


FIG. 14

the minor principal stress acting on a small element in the center of prism II. For the latter element, a major principal stress of $(b\,\gamma + 4\,c)$ will produce failure. Since the weight of the overlying soil produces a pressure $b\,\gamma$, it follows that the pressure q_o, which the footing must exert in order to produce failure according to this hypothesis, is equal to $4\,c$.

Utilizing the principle of the Swedish method of cylindrical sliding surfaces, H. D. Krey[8] found that a combination of a circular sliding surface and a wedge, as shown in Fig. 13(*b*), will give the smallest resistance and a corresponding value of $q_o = 6\,c$. The senior writer found that, by the use of a circular sliding surface only, without a wedge, a value of $q_o = 5.5\,c$ is obtained. Guthlac Wilson,[9] M. Am. Soc. C. E., determined analytically that the exact position of the center of this circle is as shown in Fig. 13(*c*) and the exact value of $q_o = 5.4\,c$.

[8] "Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes," 4th Ed., Berlin, Ernst, 1932.

[9] "The Calculation of the Bearing Capacity of Footings on Clay," by Guthlac Wilson, *Journal*, Institution of Civ. Engrs., November, 1941.

A satisfactory approach to the problem of bearing capacities of clay from an analytical point of view is by means of the theory of plasticity formulated by L. Prandtl.[10] He found that the shearing surfaces for an ideally plastic material are as shown in Fig. 13(*d*) and that, for a shear strength *c*, the failure unit load is $q_o = 5.14\,c$.

If clay were an ideally plastic material, the data from unconfined and triaxial compression tests on undisturbed samples would furnish a series of stress circles as shown in Fig. 14(*b*). In reality, the envelope obtained from a series of such tests is slightly inclined to the horizontal for homogeneous clays and is considerably inclined and curved for fissured clays and remolded, compacted clays. Unconfined compression tests on fissured clays have a tendency to produce rupture at a much smaller value than that indicated by the results of a series of triaxial tests. This is due to a premature disintegration of the specimens. If the confining pressure of a minor principal stress is acting, such as is the case in a surface-loading test or a triaxial compression test, this disintegration does not take place. For this reason, triaxial compression tests conducted at small lateral pressures are more satisfactory than unconfined compression tests for the determination of the *c*-value, which value one should use when translating results of compression tests on undisturbed clay specimens into ultimate bearing capacity for surface loading.

Compression tests on undisturbed soil cylinders are usually much less expensive than loading tests performed on sufficiently large areas. Therefore, the determination of the ultimate bearing capacity by means of cylinder compression tests is recommended. Since the allowable soil pressures commonly specified in building codes are conservative from the standpoint of safety against rupture of the clay, this indirect check of the ultimate bearing capacity is entirely satisfactory.

The problem becomes more difficult if the loaded areas are at a considerable depth below adjacent ground surface as, for example, in the case of caisson foundations. In such a case, the hydrostatic uplift of the soil may be added to the foregoing value of $5\,c$ and used as the ultimate bearing capacity. Actually, this value in such cases will be in excess of $(5\,c + \gamma\,h)$. There are no satisfactory theoretical solutions available, however.

Examples of the Application of Soil Mechanics to the Design of Building Foundations

General.—Since it is difficult at present to set up general rules for the design of foundations on soft ground, the application of principles of soil mechanics can be demonstrated best by presenting the soil conditions in a large city, describing the types of foundations there used, and comparing the settlement records of five large buildings.

Foundation Conditions in Boston.—Large areas in Greater Boston are underlain by a thick stratum of soft glacial clay. A typical soil profile is shown in Fig. 15(*a*). The organic silt is so compressible that only very light

[10] "Über die Eindringungsfestigkeit plastischer Baustoffe und die Festigkeit der Schneiden," by L. Prandtl, *Zeitschrift für angewandte Mathematik und Mechanik*, February, 1927.

buildings can be erected directly on the fill without suffering serious distress as a result of settlement. Therefore, it has been general practice to transfer building loads to the layer of hard or stiff clay below this stratum. Load

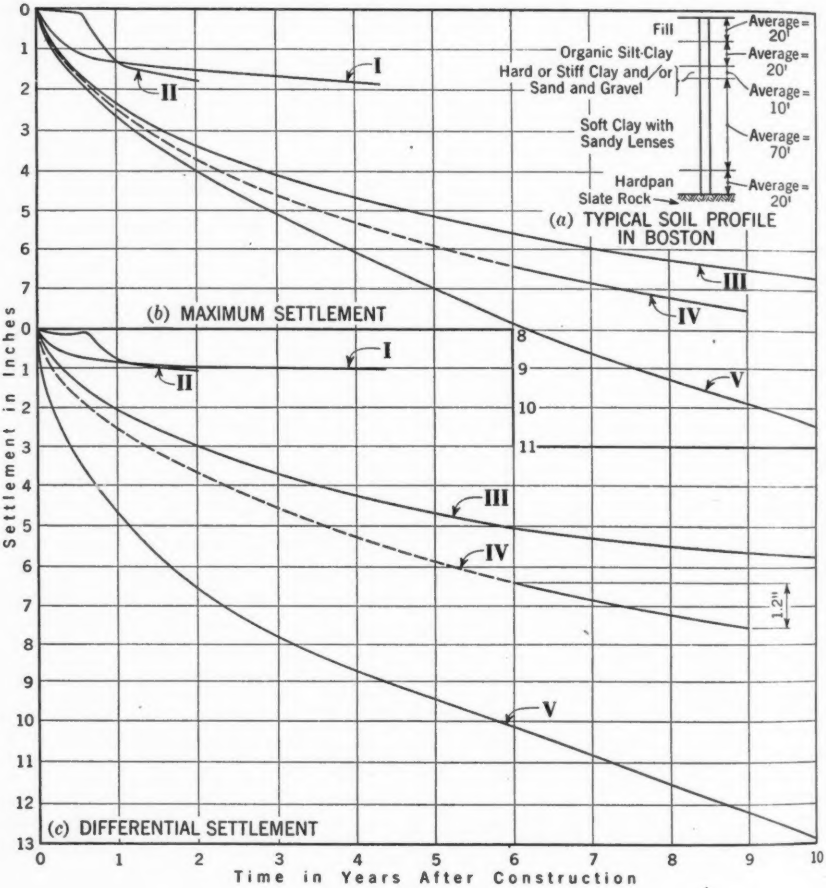


FIG. 15.—TIME-SETTLEMENT RECORDS, BOSTON, MASS.

transfer is accomplished either by means of spread footings, piles, or concrete piers with enlarged bottoms. In some areas a layer of sand instead of the hard clay is found overlying the soft clay, and in these areas piles usually have been used.

Settlements of Buildings in Boston.—The weight of soil removed to provide basement space will compensate for the weight of an average building three or four stories in height. The presence of the thick, soft, clay substratum in Boston has had little effect on buildings of this type. Larger buildings, however, have suffered from progressive and differential settlements. Such settlements have been most pronounced in cases where a building, covering

an extensive area, has produced different intensities of load on various sections of the building area. If no provision is made to equalize the net increase in pressure throughout the building area by increasing the depth of basement or sub-basement under the more heavily loaded sections, or to stiffen the structure, serious differential movements may occur.

Typical examples are shown by curves III, IV, and V in Figs. 15(*b*) and 15(*c*). The curves in Fig. 15(*b*) show the progress of movement of the point of maximum settlement in each of three different structures located within the confines of Greater Boston. The curves in Fig. 15(*c*) show the progress of differential settlement in these buildings.

The settlement represented by curves III occurred in a building supported on wood piles. These piles were driven into and, in some areas, through a stratum of sand underlain by soft clay. The thickness of soft clays varies from 55 ft to 85 ft throughout the area occupied by the building. In the most heavily loaded area, under a tower section, the average building load is equal to 1.2 tons per sq ft, and under a lighter wing of the building to 0.9 ton per sq ft. The average depth of soil removed under both sections was 10 ft. Hence, the net increase in pressure on the underlying soft clay varies between 0.6 ton per sq ft and 0.3 ton per sq ft. Furthermore, the thickness of the soft clay substratum is greatest under the more heavily loaded section. It is not surprising, therefore, that in the 10-yr period following construction the differential movement that developed between these two sections was equal to approximately 85% of the maximum settlement which occurred in the more heavily loaded section.

Curves IV correspond to another large building, for which settlement records were available only for a 3-yr period starting six years after construction. From an analysis of these data,[11] movements that had occurred prior to this period were estimated to be as shown by the dotted portions of these curves. The loads of this building are transmitted by spread footings to the top of a sand and gravel stratum, 12 ft thick. This stratum is underlain by 65 ft of soft clay. The soil conditions are comparatively uniform throughout the building area. The intensity of building load, however, varies from 2.1 tons per sq ft under a tower section to 1.2 tons per sq ft under an adjacent wing. The net increase in pressure, after deducting the weight of 15 ft of soil removed to provide basement space, is 1.4 tons per sq ft and 0.5 ton per sq ft, respectively. A contour map of the differential settlements which occurred in the 3-yr period starting six years after construction is shown in Fig. 16. (Contours represent points of equal settlement in reference to point A. The observation period equals three years, beginning six years after the erection of the building.) Curve IV in Fig. 15(*b*) shows the progress of settlement of point C in Fig. 16. Since a part of this building has not settled, this curve also represents the progress of differential movement. The rate at which this differential movement is progressing, approximately nine years after construction of the building, is 0.40 in. per yr.

[11] "Observations and Analysis of Building Settlements in Boston," by R. E. Fadum, a thesis submitted to the Faculty of the Graduate School of Eng., Harvard Univ., Cambridge, Mass., in May, 1941, in partial fulfilment of the requirements for the degree of Doctor of Science in Engineering (pp. 209–217).

A final example is shown by curves V in Figs. 15(*b*) and 15(*c*). The settlement represented by these curves occurred in a building supported by spread footings and concrete piers with enlarged bottoms bearing directly on a stratum of sand and gravel underlain by 90 ft of soft clay. The building loads in this

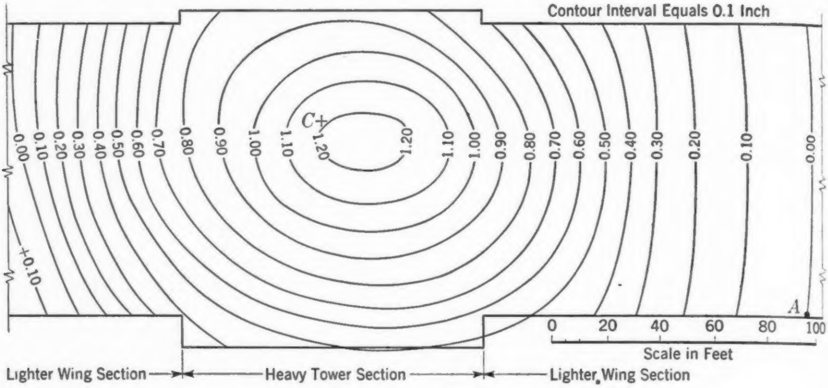


FIG. 16.—DIFFERENTIAL SETTLEMENT CONTOURS

case varied from 2.40 tons per sq ft under a tower section to 0.80 ton per sq ft under an adjacent wing. The net loads, after deducting the weight of soil removed from these areas, were 2.0 tons per sq ft and 0.7 ton per sq ft, respectively. The settlement of the tower section is shown by curve V in Fig. 15(*b*), and the differential settlement between the tower and this adjacent wing is shown by curve V in Fig. 15(*c*).

Many other striking examples of serious settlements could be cited from other cities, and, were it not for a feeling of false shame that induces engineers and architects to withold permission for publication of the records of such settlements, it would be possible to add still more. Probably the largest settlements on record are those which have occurred in Mexico City, Mexico, where settlements are recorded in units of feet rather than inches. It is not surprising, therefore, that the principle of a "floating foundation" has been incorporated there without compromise in the design of a modern building.[12]

The experience gained from a study of observed settlements indicates forcefully that, for buildings covering large areas and producing considerable differences in loading between various sections, differential settlements can be reduced to a tolerable amount by excavating a quantity of soil approximately equal in weight to the load of the superimposed structure, so that there will be practically no net increase in stress in underlying soft clay strata. Even the fulfilment of this requirement will not eliminate settlements entirely, for the following reasons:

(*a*) The removal of a large mass of soil will produce an elastic deformation of the underlying soil resulting in a slight upheaval of the bottom of the excavated pit. Upon replacing the weight of soil removed by the weight of

[12] "The Floating Foundation of the New Building for the National Lottery of Mexico," by Jose A. Cuevas, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., June, 1936.

the building, corresponding settlement will take place. Because of the confining effect of the edges and the corners of an excavated pit, this upheaval and subsequent settlement theoretically should be largest in the center of the area, approximately one half of this value along the edges and one quarter at the corners. The actual magnitude of the upheaval is dependent chiefly on the lateral reinforcement of the soft soil by sand layers and lenses. Although it is known that this lateral deformation can assume large proportions for more homogeneous deposits of very soft clay, such as those which exist in Mexico City, no serious movements have ever been observed in Boston. This upheaval was measured, as will be described subsequently, during construction of a large building in Boston.

(*b*) Since building loads are usually transferred on to a bearing stratum by means of individual column footings, wall footings, caissons, or pile clusters which cover only part of an entire building area, loads concentrated on this stratum will cause a slight deformation and consolidation of the soil immediately below the footings. Again the amount of settlement due to this cause will be reduced along the edges and particularly at the corners of a building area, due to the increased lateral confinement of the soil at these places.

(*c*) If the bearing stratum is very thin, there will be a net increase in stress in limited volumes of the softer soil underlying the bearing stratum. The soil will adjust itself to these increased stresses chiefly by consolidation and to a small degree also by lateral deformation.

The experience of recent years confirms the requirements for the design of building foundations which one of the writers expressed [3] in 1932 in the following conclusion:

> "* * * the whole problem of building foundations on clay boils down to these two simple principles: first, do not disturb the natural structure of the clay; if you do, no human being is able to restore its original strength; second, decide on a certain rate of settlement which you do not wish to exceed, and determine that pressure which will cause this rate of settlement; the difference between the building load and the above pressure is the weight of soil which must be removed before erecting the building * * *."

Special attention should be given to the magnitude of the differential settlements, which are dependent on the design of the building, the soil conditions, and inequalities in the preconsolidation load. In addition, it should be noted that the allowable magnitude, rate, and distribution of settlement depend to a large extent on the type of building. It is obvious that a masonry facing consisting of 4-ft by 6-ft limestone blocks, with very thin joints, is many times more sensitive to differential settlements than, for example, a brick building.

The Liberty Mutual Insurance Company Building in Boston.—An example of the application of soil mechanics to the design of the foundations for a typical office building is furnished by the Liberty Mutual Insurance Company Building which was built in 1936–1937. This building has a plan area 270 ft by 116 ft, and consists of a heavy tower section and two lighter wings. It is faced with large, thin, limestone blocks which are very sensitive to differential movements. The basement was so designed that the weight of the soil excavated was slightly

less than the building load on all sections of the plan area occupied by the building.

The building load is transferred to the basement by a great number of columns, which in turn are transferred by concrete piers on to the surface of a hard clay stratum at a depth of about 40 ft below ground surface. The bottoms of the piers are increased in area by means of bells, the diameters of which are

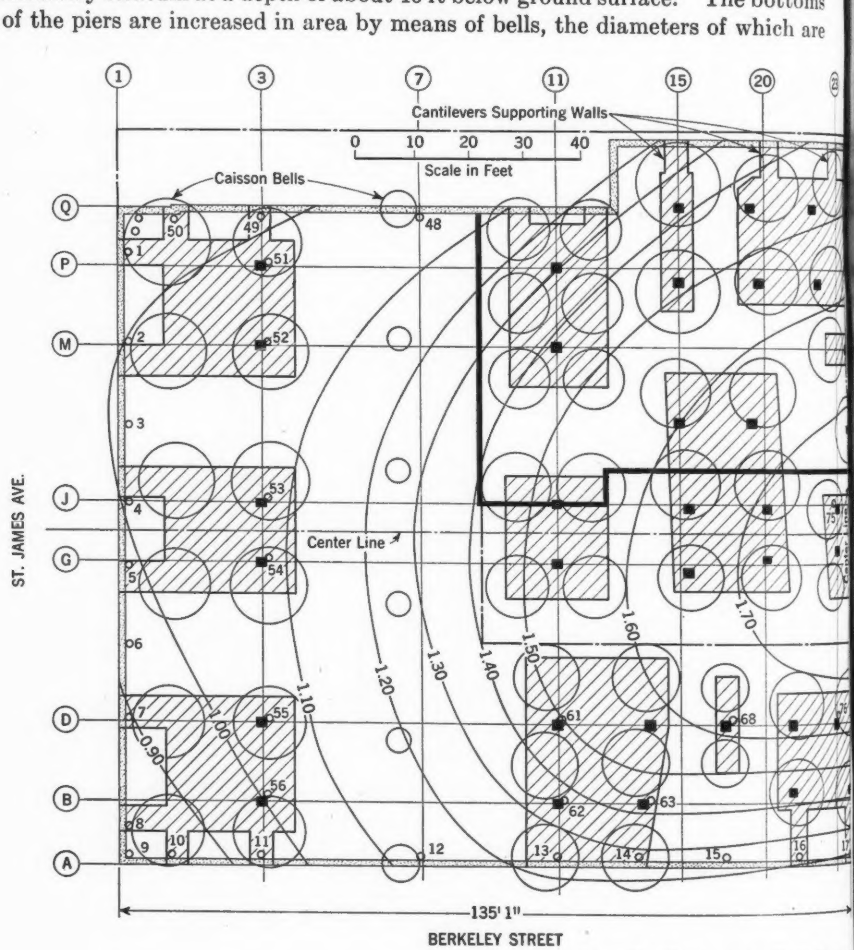
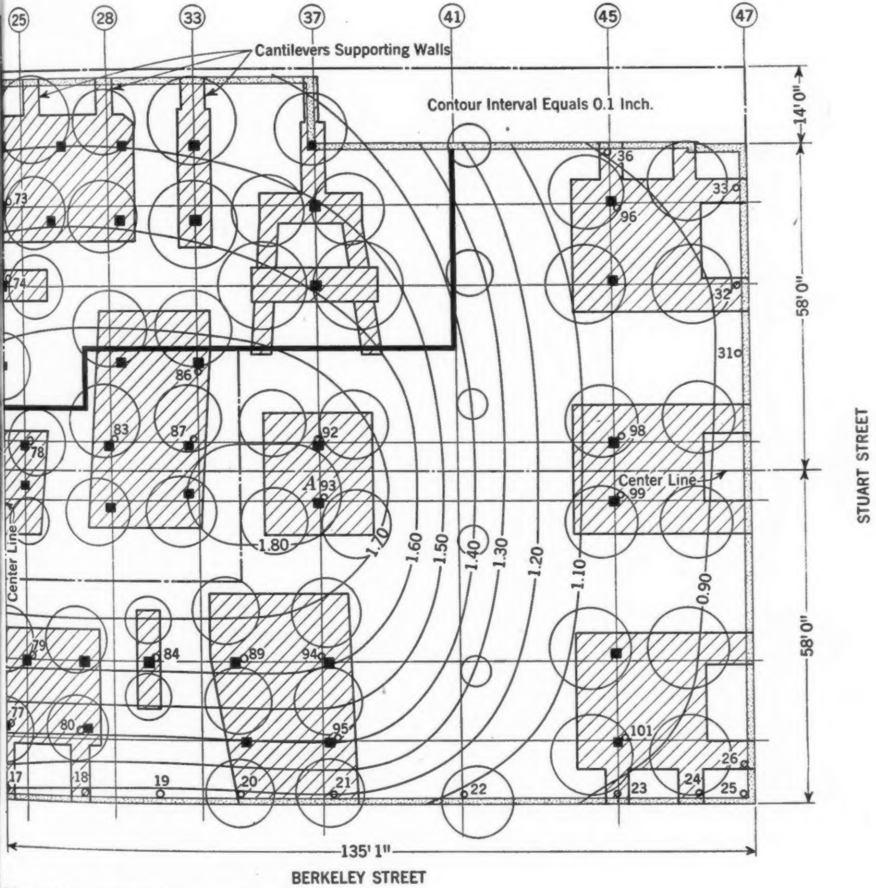


FIG. 17.—FOUNDATION PLAN, LIBERT

governed by the column loads as well as by the thickness of the underlying hard clay stratum. The allowable load on the hard clay was determined from compression tests on undisturbed samples. The hard clay, which averages 10 ft in thickness, is underlain by 60 ft of soft clay.

To reduce the differential settlement of the exterior foundation walls to a minimum, they were designed as heavily reinforced girders. To reduce the differential settlement between the exterior foundation walls and the interior

footings, these walls are supported by cantilever beams which transfer the wall loads to piers carrying interior columns. This arrangement increases the absolute settlement of the circumferential wall, but in so doing it decreases the differential movement in reference to the interior of the building. Only at those points where it was not possible to arrange a cantilever system were piers placed directly beneath the walls.



Mutual Insurance Company Building

The foundation plan for this building is shown in Fig. 17. The small dark squares represent interior column locations, the shaded areas represent mats, and the circles represent the diameters of pier bells bearing on the stratum of hard clay. The contours represent points of equal settlement in reference to a permanent bench mark for a period of 4.3 years.

The building load varies from 1.3 tons per sq ft under the central tower section to 0.8 ton per sq ft under the adjacent wings. To equalize the net

increase of pressure throughout the building area, a sub-basement was installed under the tower section. Thus, the net increase in pressure was made equal to 0.2 ton per sq ft in all areas.

The small numbered circles in Fig. 17 designate the locations of permanent settlement observation points which were established during the construction of the substructure. The elevations of these points were determined periodically by means of precision water-level surveys in reference to a permanent bench mark that was installed in one corner of the basement. The bench mark was installed by driving a pipe through an exterior permanent casing into hardpan directly overlying bedrock to a total depth of 120 ft below the original ground surface. The experience gained from conducting level surveys with this water-level precision instrument[13] has shown that with reasonable care a line of levels can be made around a closed loop containing about twenty observation points with an average error of closure of 0.012 in. Such a level survey can be made without difficulty in basements that are fully utilized for storage purposes.

As stated, the contours in Fig. 17 show the lines of equal settlement. These settlements occurred in a period of 4.3 years following installation of the observation points. The time-settlement curve for the point marked A, at which the maximum settlement has been observed, is shown by curve I in Fig. 15(*b*). This curve shows a rate of increase, 4.3 years after construction, of only 0.12 in. per yr, as compared with a rate of 0.40 in. per yr observed some nine years after construction in a building, previously mentioned, of similar weight and size. The maximum differential settlement at this time is slightly in excess of 1.0 in., and the rate of increase as shown by curve I in Fig. 15(*c*) is negligible.

The New England Mutual Life Insurance Company Building.—A more difficult foundation problem than that which was encountered in the case just described presented itself in connection with the design of the foundations for the New England Mutual Life Insurance Company Building. The subsoil conditions are very similar in both cases. However, the following complications had to be considered in the latter case: Two sections of the building area were pre-loaded by two fairly heavy buildings that were removed just prior to the construction of the new building. An equal area between these buildings, where the heaviest section of the proposed building was to be located, had never before been loaded. The building was to cover an area of 340 ft by 200 ft and was to consist of a heavy ten-story section occupying a "T-shaped" area, and one one-story, two two-story, and two four-story sections. The one-story and four-story sections, at some future time, might be built up to the full height of the heavy ten-story section. The foundations had to be designed to provide not only for the first stage of construction, but also for all enlargements contemplated for the future.

Extensive underground explorations were undertaken. Nine exploratory borings were made from which almost continuous thin-walled tube samples

[13] The type of instrument used in these surveys is described in "Practical Application of Soil Mechanics: Settlement of Structures in Europe and Methods of Observations," by Karl Terzaghi, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), pp. 1432–1448, particularly pp. 1433–1436.

were obtained from the hard and soft clay strata. Two large-diameter borings were then made for the purpose of obtaining undisturbed samples for laboratory investigations of the compressibility characteristics of the material below the elevation at which the building was to be founded.

Settlement investigations for various preliminary designs of the building indicated that, with the aforementioned complications, serious differential settlements could be avoided only by using the principle of a floating foundation, thereby excavating as much soil as the building, including the anticipated future enlargements, would weigh. This estimated weight was 130,000 tons as compared with 40,000 tons for the Liberty Mutual Building.

The weight of soil to be removed, necessary to counterbalance the load of the building in all sections of the area occupied by the building, required excavating to an average depth of approximately 35 ft. For this reason it became apparent that a sub-basement, which was not originally contemplated by the owners, would be necessary. The suggestion of such a sub-basement, however, was welcomed as an area for garaging automobiles, since the parking difficulties in this section of the city are particularly acute.

The only other feasible solution of this foundation problem would have been to transfer the entire building load by means of piles or piers on to the hardpan underlying the soft clay stratum at a depth of at least 125 ft beneath the original ground surface. A preliminary comparison of the cost of this type of foundation with a floating type of foundation showed that the transfer of the load to the hardpan would be considerably more expensive.

The design of the foundation was influenced vitally by the fact that the functional requirements called for clear working spaces, free of columns. This type of load distribution lent itself to the design shown in Fig. 18(*a*). The circumferential lines and the lines EF, GH, IJ, KL, MN, and OP represent the main walls of the building which carry the major part of the building load. These walls are extended through the sub-basement and the basement and are heavily reinforced to provide sufficient stiffness to reduce differential settlements that might result from irregularities in thickness and consistency of the hard clay stratum. It was determined, from load tests and unconfined and triaxial compression tests, that the allowable load on the hard clay should not exceed 4 tons per sq ft. The widths of the wall footings as shown by the dotted lines in the foundation plan, Fig. 18(*a*) are such that this unit load is not exceeded. The bearing pressure under the circumferential walls was increased in relation to the unit load for the interior wall footings to reduce the differential settlements between the outside and the inside walls. An additional increase in the design unit load on the hard clay was provided at the corners of the building by decreasing the footing widths at these locations.

Typical cross sections, showing the sections of the walls, wall footings, and basement slabs, are given in Figs. 18(*b*), 18(*c*), and 18(*d*).

The uplift produced by a head of 22 ft of water controlled the design of the sub-basement slab in most instances. Exceptions to this are found in those areas where columns and heavy equipment, such as boilers, rest directly on the slab. Thicker slabs were necessary to provide for these loads. The column loads are not carried directly down to the hard clay, but are transferred

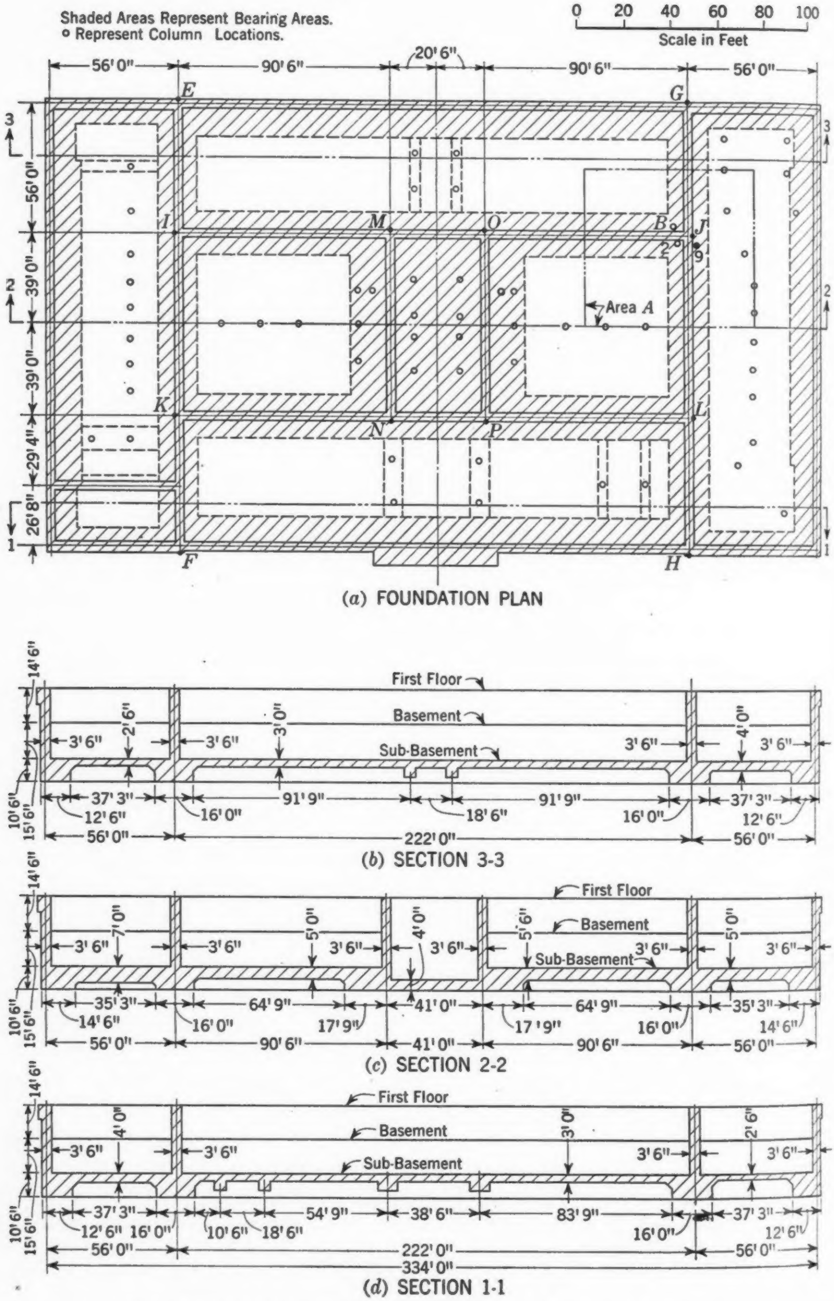


FIG. 18.—NEW ENGLAND MUTUAL LIFE INSURANCE COMPANY BUILDING

to the main walls for the purpose of eliminating serious differential movements between the individual columns and the main walls. The objection might be raised that a more economical transfer of load might have been arranged by the use of individual girders beneath the sub-basement floor. Unfortunately, the depth of girders required would have been such that the girders would have borne on the hard clay and thereby defeated the purpose of the load transfer. With the exception of the small rectangular area bounded by MNOP in Fig. 18(*a*), all of the sub-basement floors are supported on the main walls and are separated from the hard clay by at least 3 ft of the highly compressible, organic silt-clay. It was specified that, if it were discovered, during the progress of construction, that the hard clay in some areas approached the bottom of the sub-basement slabs, it would be necessary to excavate to a sufficient depth and replace the hard clay by soft organic silt-clay.

The unsymmetrical footings of the circumferential walls, shown in Fig. 19(*b*) evolved from adapting considerations of soil mechanics and requirements for simplicity and economy of construction to the conventional type of footing

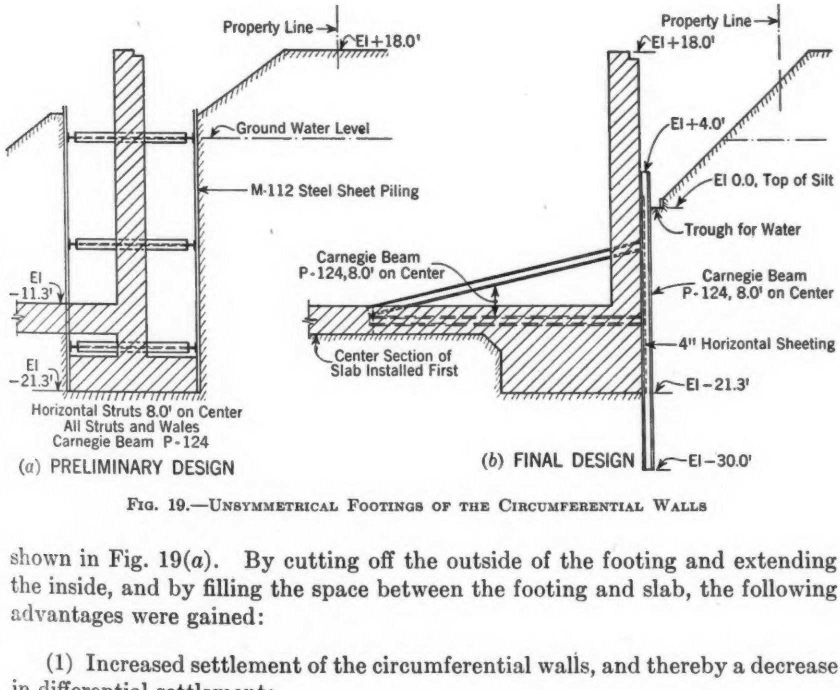


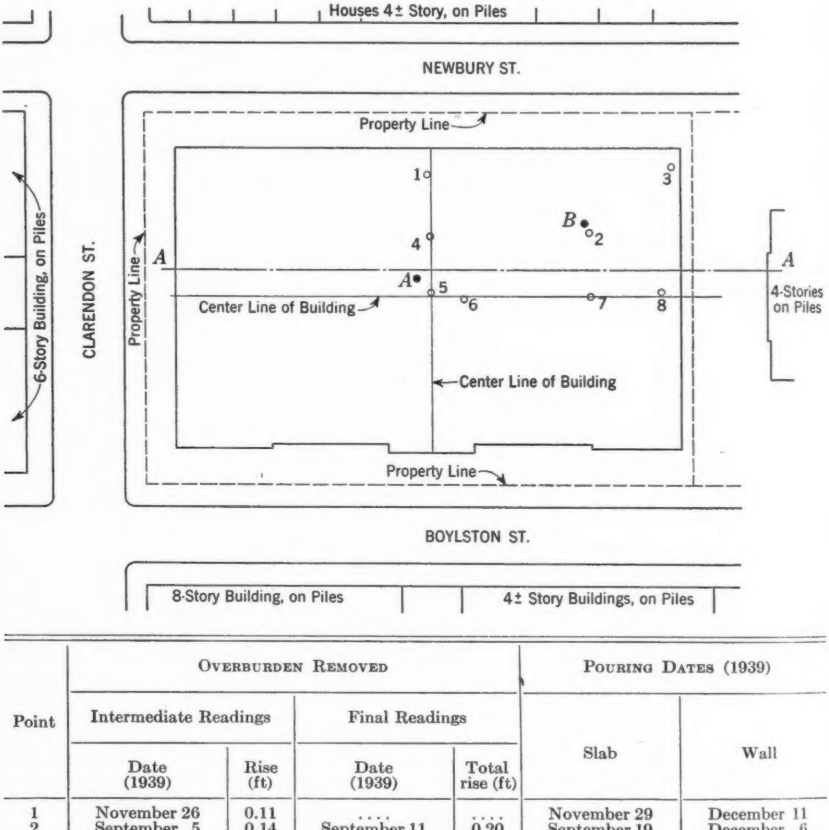
FIG. 19.—UNSYMMETRICAL FOOTINGS OF THE CIRCUMFERENTIAL WALLS

shown in Fig. 19(*a*). By cutting off the outside of the footing and extending the inside, and by filling the space between the footing and slab, the following advantages were gained:

(1) Increased settlement of the circumferential walls, and thereby a decrease in differential settlement;
(2) Material reduction in the span of the sub-basement slabs; and
(3) A simple and economical method of construction.

The general excavation was carried first to a depth of about 20 ft, to the top of the organic silt-clay, and from there vertical H-section piles, as shown

in Fig. 19(*b*), were driven. As the excavating proceeded, horizontal and inclined H-section braces were installed. These bore against sections of the slab that had been installed prior to excavating the final berm. The horizontal braces were later concreted into the slab, and the inclined braces were cut off.



Point	Overburden Removed				Pouring Dates (1939)	
	Intermediate Readings		Final Readings		Slab	Wall
	Date (1939)	Rise (ft)	Date (1939)	Total rise (ft)		
1	November 26	0.11	....	....	November 29	December 11
2	September 5	0.14	September 11	0.20	September 19	December 6
3	October 24	0.16	October 30	0.19	November 2	December 1
4	October 12	0.20	October 23	0.22	October 27	December 26
5	October 12	0.24	....	....	October 27	December 26
6	October 19	0.20	October 23	0.23	October 27	December 26
7	September 5	0.16	September 11	0.21	September 22	November 27
8	October 24	0.15	October 30	0.19	November 2	December 1

Points 1 to 8 were established in the ground prior to the removal of the overburden (June, 1939). Overburden (wet) = 2.24 tons per sq ft.

Fig. 20.—Foundation Movements, New England Mutual Life Insurance Company Building

The vertical piles and horizontal wooden sheeting remained in place and were used as the outside form work. The development of this ingenious solution is essentially due to L. S. Homer, M. Am. Soc. C. E., superintendent for the Turner Construction Company.

Observations of Movements.—It was stated that, if a foundation is designed in such a manner as to produce a zero net increase in stress in underlying compressible soil after excavating and building operations are completed, the

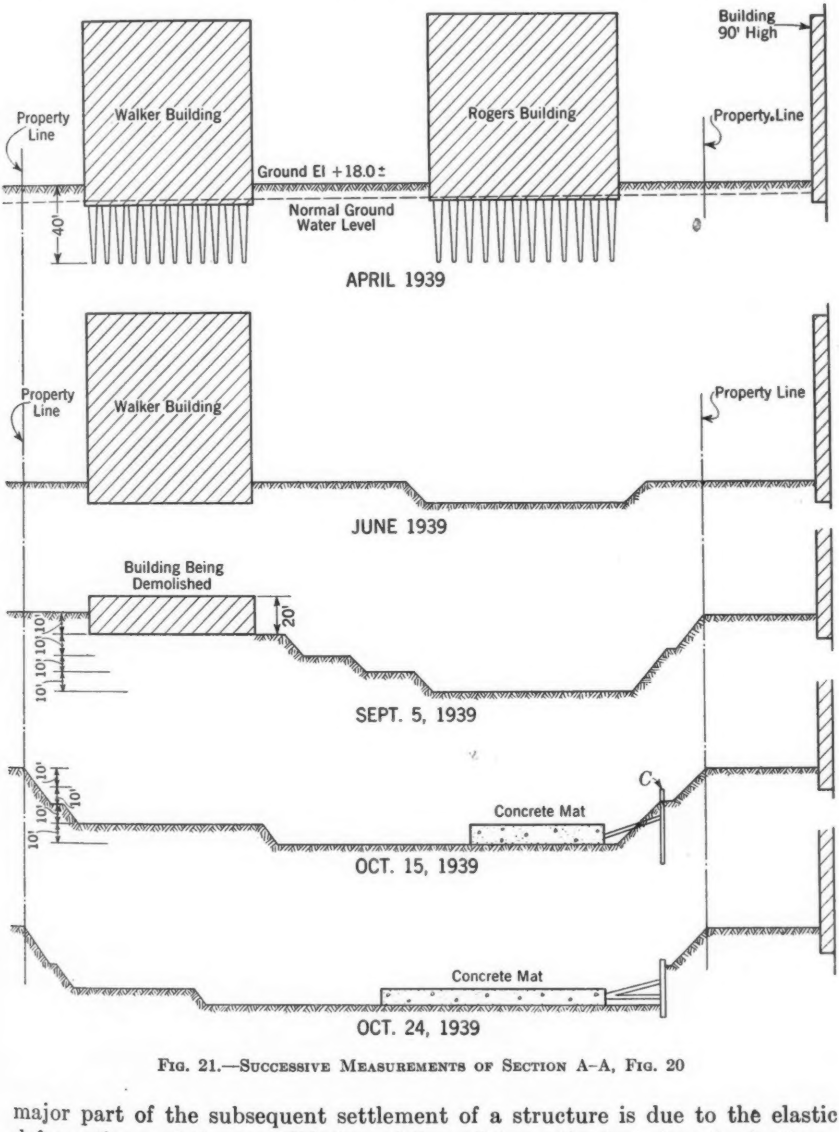


FIG. 21.—SUCCESSIVE MEASUREMENTS OF SECTION A–A, FIG. 20

major part of the subsequent settlement of a structure is due to the elastic deformation and the consolidation of the soil immediately below the footings. Were it possible to distribute the building load uniformly over the building area, and were it possible to excavate and apply the building load simultane-

ously, no settlement would result. However, it is impracticable to design and construct a building in such a manner that these requirements are fulfilled, and it is necessary, therefore, to investigate the consequences that might result if these two conditions are not fulfilled.

For this purpose an effort was made to obtain a record of the movement of the bearing stratum during and subsequent to all construction operations. To obtain this record, it was necessary to measure the rise in elevation of this stratum, resulting from the removal of an average of 35 ft of overburden, and then to determine the downward movement which took place when the weight of the foundations and superstructure was added.

Prior to excavating, underground reference points were established at eight locations in one quadrant of the building area, as shown in Figs. 20 and 21. These reference points consisted of 2-in. tubes, each 6 ft in length, which were driven into the hard clay. This was accomplished at each location by advancing the casing to the top of the hard clay. After the casing had been cleaned out, a steel tube was lowered and driven 6 ft. The elevation of the top of the tube was then established. For the purpose of marking the location of each tube, colored wooden sticks, which were weighted to prevent flotation, were lowered as the casing was withdrawn.

The elevations recorded for each point are given in the table in Fig. 20. The maximum upward movement observed when the eight points were first uncovered was 2.9 in., at point 5 in the center of the area, and the minimum was 1.3 in., at point 1, which is located on the shorter axis near the periphery of the building area. The second set of observations, which was made on six of the points one week after they were first uncovered, and just prior to installing and sub-basement slab, showed that in this period they had continued to rise so that the ultimate total rise for each point was approximately the same.

Settlements resulting from the placing of wall footings 10 ft thick and adjacent sub-basement slabs were determined at the location marked "B" in Fig. 20 by observing the movement of the top of a $1\frac{1}{4}$-in. steel rod which had been driven 3 ft into the hard clay. This rod, which projected up through a piece of pipe encased in the concrete, settled approximately 0.5 in. during the placing of the concrete. Subsequent observations which were made at periodic intervals in reference to a permanent bench mark (of the type installed in the Liberty Mutual Insurance Company Building) showed that this point moved downward as follows:

Time of observation	Approximate load (tons per sq ft)	Total downward movement (inches)
As concrete was placed	0.75	0.5
After six days	0.75	0.75
After one month	1.00	1.00
After two and one-half months	1.25	1.00
After three months	1.75	1.25

—a total of $1\frac{1}{4}$ in. during the three-month period following the installation of the concrete slab and footings at this location. At the end of this three-month period, during the installation of the basement walls, permanent observation

points were established at seventy-three locations throughout the sub-basement area. The elevations of these points were observed periodically in reference to the permanent bench mark by means of precision water-level surveys. Point 9 was located within 10 ft of the locations of underground observation points 2 and B, thus affording a means of continuing observations of the movement of the bearing stratum at this location.

At point A, Fig. 20, a 4-in. boring had been made in April, 1938. When the overburden was removed in October, 1939, this hole was found to have been reduced to 2.5 in.

The top of a pile at driven point C (see Fig. 21) was observed in October, 1939, to have moved inward $\frac{3}{4}$ in. and to have been raised $\frac{1}{4}$ in. In December, 1939, after the exterior footings and walls had been poured, this pile had moved downward $\frac{3}{4}$ in.

After the excavation and shoring were completed, the street curbs moved in about $1\frac{1}{4}$ in. and down about $1\frac{1}{4}$ in. No horizontal or vertical movement was observed in adjacent buildings.

A continuous record of the movements that occurred during the construction period, following the demolition of buildings which formerly occupied the site, together with diagrams which show the change in net unit load at this location, are shown in Fig. 22. The shaded diagram represents the net unit load on

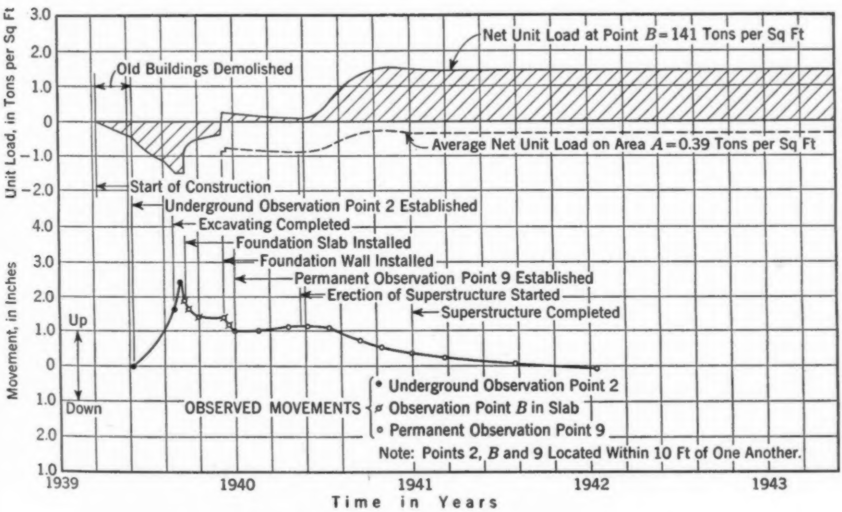


FIG. 22.—TIME-LOAD DIAGRAMS AND OBSERVED MOVEMENTS

the bearing stratum at the location of the intersection of two interior cross-wall footings (see Fig. 18(*a*)) where points 2, B, and 9 are located. The dotted diagram shows the average unit load on the area marked "A" in Fig. 18(*a*). This area includes one half of the unloaded area adjacent to these wall footings. The ordinates of both diagrams show the net change in unit load resulting from all construction operations—that is, the algebraic sum of the unit load

removed by the demolition of old buildings, the unit load removed by excavating an average of 35 ft of soil, the temporary unit load added by the lowering of the ground-water table during construction, and the unit load of the foundations and superstructure of the new building.

The shaded diagram shows an ultimate local increase in pressure immediately below the wall footings. The dotted diagram, which represents the average change in unit load for a typical section of the building area, including unloaded parts, shows an ultimate net decrease at the conclusion of all construction operations. Therefore, the subsoil is subject to an increase in pressure only to a limited depth below the footings, and for this reason only a comparatively small volume of soil will be compressed.

The loading diagrams assist in an interpretation of the curve showing the foundation movements. This latter curve shows that during the excavating period the bearing stratum rose 2.40 in. After the foundations had been installed, it settled back 1.40 in. In the six-month period that followed, during which time hydrostatic pressure was increasing under the slab with very little building weight being added, a slight rise in elevation took place. Then, as the building load increased rather rapidly during the erection of the superstructure, the stratum began to settle. As of January 15, 1942, which

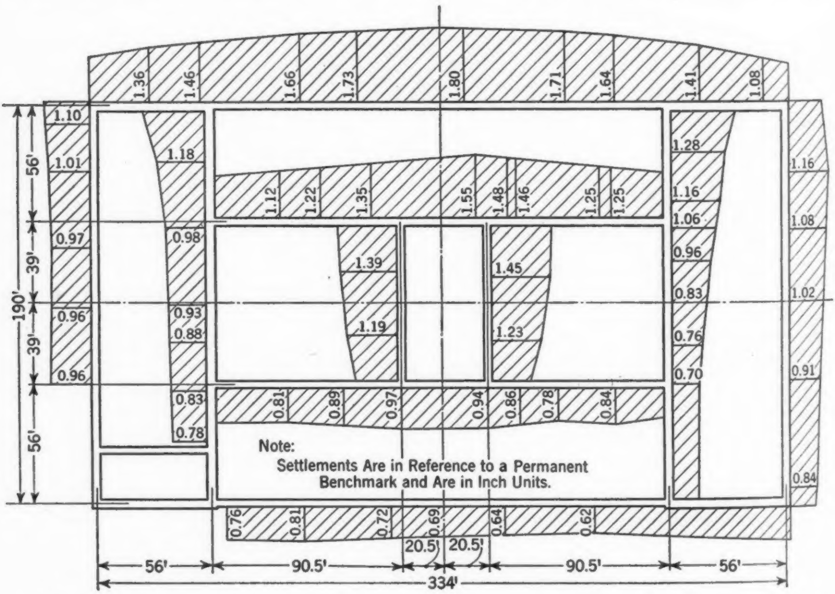


Fig. 23.—Settlement Profiles, New England Mutual Life Insurance Company Building

was approximately one year after the superstructure was substantially completed, the net movement was 0.06 in. downward. It will be noted that, of the total downward movement (2.46 in.), 1.40 in. occurred during construction of the foundations prior to erection of the superstructure.

The movements that have occurred at other points throughout the building area, as determined from periodic precision water-level surveys of the permanent observation points, are shown by the settlement profiles in Fig. 23. The settlements that are represented by these profiles have occurred in the 2-yr period following the installation of the foundation walls. The settlements thus represented include the movements which took place during erection of the superstructure. These profiles show little differential movement.

The progress of settlement of the point at which the maximum settlement has occurred is shown by curve II in Fig. 15(*b*). The maximum differential settlement determined from the seventy-three points under observation is shown by curve II in Fig. 15(*c*). The trend of these curves is sufficiently well established to indicate that future settlements will be of a small order of magnitude.

Conclusions

(1) The analysis of observed settlements of buildings in Boston shows clearly that settlements can be reduced to a tolerable magnitude by removing a quantity of soil which, in weight, is approximately equal to that of the superimposed structure.

(2) This method of "floating" a building is particularly adaptable to locations where a suitable bearing stratum is located from 20 to 40 ft below the ground surface and where soil and ground-water conditions permit the necessary excavating.

(3) A structure that is underlain by soft soil, and which obtains support from such soil, will undergo some settlement even though the weight of the structure does not exceed the weight of soil removed. The removal of a mass of soil will cause an upheaval of the bottom of an excavated pit, due in part to a deformation and in part to a swelling of the underlying soil. Upon reestablishing the original stress conditions, a settlement at least equal to the magnitude of the upheaval will take place.

(4) The settlements due to deformation and recompression following swelling take place essentially during the construction period. Progressive settlements will occur subsequent to this period for many years at a decreasing rate if the weight of the structure is greater than the weight removed from the building area prior to its construction.

(5) Differential settlements are best controlled by designing the foundation and the superstructure as an integral unit, with special attention to stiffness. Where this is not acceptable, design for stiffness must be confined to the foundation, which increases the cost and decreases its effectiveness.

(6) Differential settlements can be controlled to some extent by increasing the unit loading on those areas where the smallest progressive settlements are anticipated and decreasing the unit loading where the largest progressive settlements are anticipated.

Acknowledgments

The writers wish to acknowledge gratefully the cooperation of Chester Lindsay Churchill, architect for the Liberty Mutual Insurance Company

Building; C. N. Godfrey of the architectural firm of Cram and Ferguson, which designed the New England Mutual Life Insurance Company Building; the Turner Construction Company, and in particular L. S. Homer, superintendent of construction for both the Liberty Mutual and the New England Mutual Insurance Company buildings; H. A. Mohr, M. Am. Soc. C. E., of the Raymond Concrete Pile Company; and the City of Boston Building Department. Messrs. P. C. Rutledge, F. M. Baron, Assoc. M. Am. Soc. C. E., and E. M. Fucik and K. S. Lane, Juniors, Am. Soc. C. E., former members of the staff of the Graduate School of Engineering, Harvard University, Cambridge, Mass., assisted the writers in the design of the foundations of these buildings. J. A. Delaney, a present member of the staff, assisted in conducting level surveys. M. J. Hvorslev, Assoc. M. Am. Soc. C. E., supplied copy for Fig. 6; and reproduction of the information in Figs. 20 and 21 was made possible through the courtesy of the Turner Construction Company.

APPENDIX

Notation

The following letter symbols, used in this paper, conform essentially to "Soil Mechanics Nomenclature"[14] presented in 1941 by the Committee of the Soil Mechanics and Foundations Division on Glossary of Terms and Definitions and on Soil Classification:

A = area covered by a footing;
b = width of a footing;
c = a constant shear strength;
e = void ratio;
h = depth of a footing;
P = concentrated, or equivalent concentrated, load on a footing;
p = average unit load on a footing;
q_o = average unit pressure exerted by a footing;
$R = \sqrt{r^2 + z^2}$;
r = radius or radial distances from an axis;
z = distance parallel to the z-axis (depths), measured from the base of the footing;
α = a concentration factor (Eq. 1*b*);
β = an angle in Fig. 10;
γ = weight of soil per unit volume;
Δ = increment of increase; thus: Δp = any additional unit pressure; Δe = change in void ratio, etc ;
δ = unit strain; and
σ = principal stress.

[14] *Manual of Engineering Practice No. 22*, Am. Soc. C. E., 1941.

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PAPERS

FLOW CHARACTERISTICS AT RECTANGULAR OPEN-CHANNEL JUNCTIONS

By Edward H. Taylor,[1] Jun. Am. Soc. C. E.

Synopsis

The research of which this paper is a part comprised a study of the phenomena associated with the combining and dividing flow of water in open and closed conduits. The paper is concerned only with the open-channel problem, in which the variables involved are so numerous that only an empirical solution seems possible. The conformity of actual behavior to theory is discussed, and an empirical solution is included which may be applied to rectangular channels. For other prismatic channels or for natural streams, the flow characteristics at the junction are best determined by models controlled in an hydraulic laboratory.

The original data and reports are on file in the Engineering Societies Library[1a] and at the University of California in Berkeley.

Notation

The letter symbols introduced in this paper are defined where the first appear and are assembled for convenience of reference in the Appendix.

Combining Flow

The principal problem of combining flow may be stated as follows: When two streams combine in a single channel, the depth just below the junction will be fixed by the backwater characteristics of that channel and the magnitudes of the combined rates of flow. The problem is to predict the depth in each tributary channel just upstream from the junction. One of the factors involved is the ratio in which the incoming flows are divided. In studying the combining streams, this ratio was made one of the independent variables.

Note—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by April 1, 1943.

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It is immediately apparent that an unlimited number of different types of stream intersections could be devised. The angles of intersection, the breadths of the channels, the directions of flow, etc., could be combined in such a multiplicity of ways as to make a completely exhaustive investigation a matter of many years' work. The studies have indicated that generalization of the results presented is not possible, or even desirable, so that no attempt has been made to present a mathematical statement applicable to every type of stream intersection. Rather, the attempt has been to investigate the importance of the problem, to point out limitations in the theory, and to demonstrate how the results may be applied in practice.

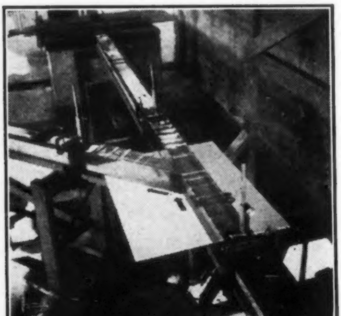


FIG. 1

Experimental Equipment.—The tests were conducted in small, horizontal, rectangular channels, all 4 in. wide and with a maximum depth of 4 in. The intersections were constructed of a transparent plastic, and the remainder of the channels were built of galvanized sheet metal. Fig. 1 gives a general view of the experimental setup. Water-surface elevations were measured with point gages, and rates of flow by calibrated diaphragm orifices in the supply pipe. The elevations of the water surfaces in both channels could be varied by means of adjustable gates at the lower ends.

Theory and Assumptions.—The scope of discussion will be limited to the case shown in Fig. 2. The restrictions are: (*a*) The channels are of equal width, (*b*) the bottom slopes are all zero, (*c*) the flow is from channels 1 and 2 into channel 3, and (*d*) channels 1 and 3 lie in a straight line. The assumptions are: (1) The flow is parallel to the channel walls immediately above and below the junction, (2) ordinary wall friction is negligible in comparison with other forces involved, and (3) the depths in channels 1 and 2 are equal immediately above the junction.

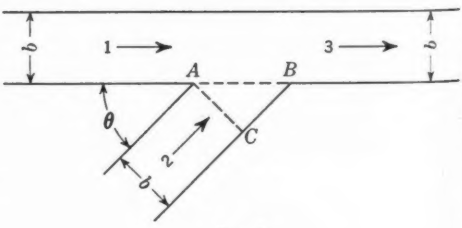


FIG. 2

From the principle that the net force acting upon a fluid system is equal to the rate at which the momentum of the system is changed, the following equations are obtained:

The net force, F, acting in the direction 1 to 3 is—

$$F_{1-3} = \frac{\gamma\, b\, (y_a)^2}{2} + \frac{\gamma\, b\, (y_a)^2}{2} \cos\theta - \frac{\gamma\, b\, (y_b)^2}{2} + U \quad \ldots\ldots\ldots (1)$$

in which: γ = specific weight of water; b = width of a rectangular channel section; y_a = depth of flow above the junction; y_b = depth of flow below the junction; θ = angle between the diverging channels; and U = an unknown reaction. The rate, ΔM_{1-3}, at which momentum is changed in the direction 1 to 3 is—

$$\Delta M_{1-3} = \frac{Q_3 \gamma V_3}{g} - \frac{Q_1 \gamma V_1}{g} - \frac{Q_2 \gamma V_2}{g} \cos \theta \quad \ldots\ldots\ldots\ldots (2)$$

in which: Q = rate of flow, the subscript referring to the appropriate channel; V = average velocity in the channel denoted by an appropriate subscript; and g = acceleration of gravity.

Upon examining Eq. 1 more closely, it is apparent that U must be the component of the pressure force upon the part of the wall marked BC in Fig. 2. If a further simplification is made so that the depth in the triangle ABC is everywhere equal to y_a, then U evidently must be equal and opposite to the term $\frac{\gamma\, b\, (y_a)^2}{2} \cos \theta$. Eq. 1 is now simply:

$$F_{1-3} = \frac{\gamma\, b\, (y_a)^2}{2} - \frac{\gamma\, b\, (y_b)^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Equating Eqs. 1 and 2:

$$\frac{Q_1 \gamma}{g} V_1 + \frac{\gamma\, b\, (y_a)^2}{2} + \frac{Q_2 \gamma}{g} V_2 \cos \theta = \frac{Q_3 \gamma}{g} V_3 + \frac{\gamma\, b\, (y_b)^2}{2} \quad \ldots\ldots (4)$$

Eq. 4 may be put in dimensionless form by the introduction of the factors n_q, n_y, and k_3. The final form is:

$$(n_q)^2 (1 + \cos \theta) - 2\, n_q + 1 = \left[\frac{1 - (n_y)^2}{4\, k_3} + 1 \right] n_y \quad \ldots\ldots\ldots (5)$$

Experimental Results.—The factor k_3 in Eq. 5 varies inversely as the cube of the depth y_b. As this depth was difficult to measure, owing to the rough water surface below the junction, the numerical value of the factor k_3 as obtained experimentally was of doubtful accuracy. Therefore, Eq. 5 was modified so that the value of k_2, or the ratio of the velocity head to the depth in the branch channel, was made the independent variable. The resulting equation is:

$$k_2 = \frac{(n_q)^2 [(n_y)^2 - 1]}{4\, (n_y)^2 [2\, n_q - (n_q)^2 (1 + \cos \theta) + n_y - 1]} \quad \ldots\ldots\ldots\ldots (6)$$

Eq. 6 is shown graphically in Fig. 3 for intersection angles of 45° and 135°, and for values of n_q equal to 0.4, 0.6, and 0.8. Plotted thereon are a few of the points obtained in the laboratory. The agreement with the theory is seen to be quite good for θ = 45° (Fig. 3(*a*)) and rather poor for θ = 135° (Fig. 3(*b*)).

Summary; Combining Flow.—(1) The agreement between theory and experiment for the 45° junction supports the conclusion that all the aforementioned assumptions are justified in this case of combining flow;

(2) Since the boundary friction was the same in both cases, it may be considered as a negligible influence in comparison with the other forces;

(3) The experimental data clearly showed that the depths in the two channels upstream from the junction had nearly the same value, regardless of the angle of intersection; and

(4) The lack of agreement between theory and experiment for the 135° intersection is due to the distortion of the velocity distribution below the junction and to the fact that the flow does not remain parallel to the channel

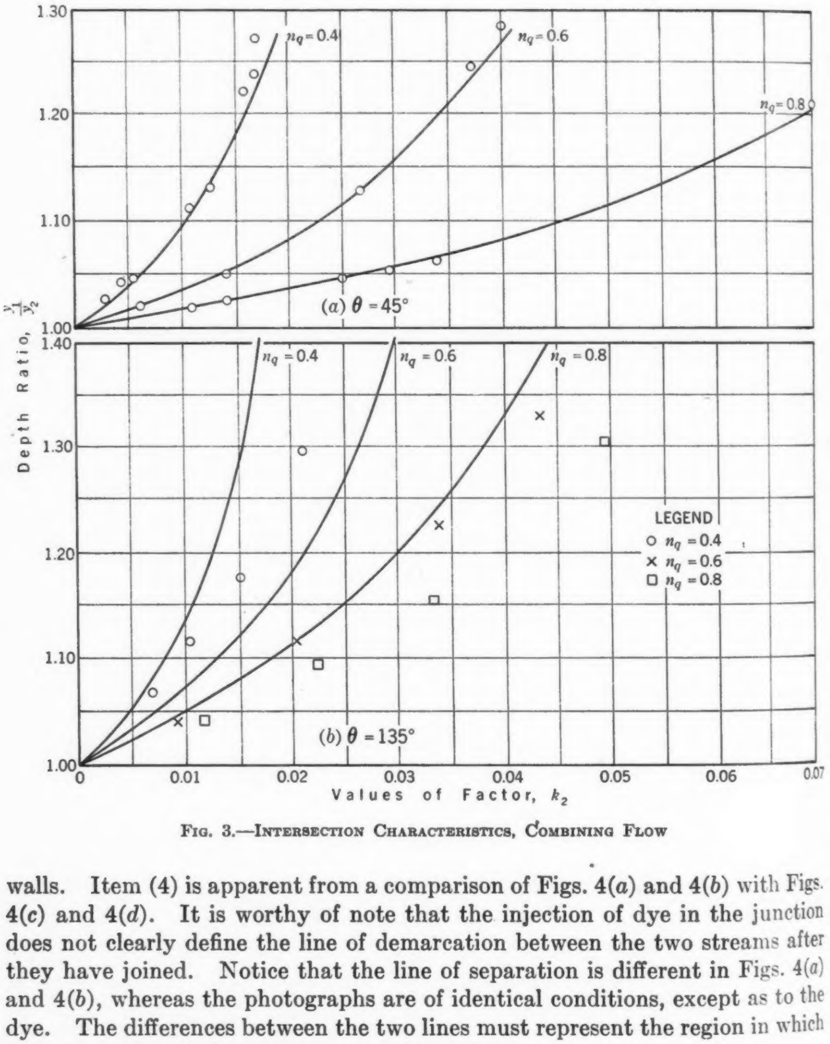
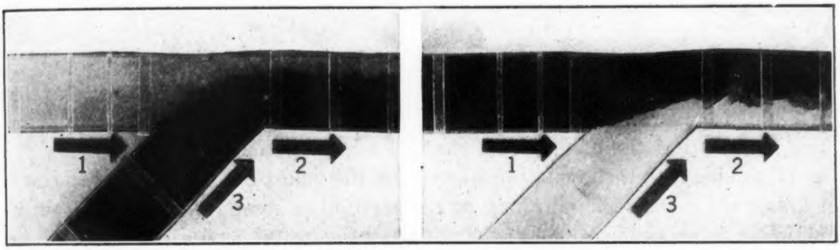


FIG. 3.—INTERSECTION CHARACTERISTICS, COMBINING FLOW

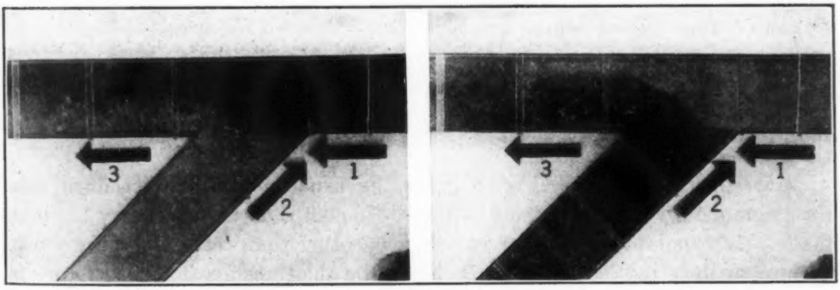
walls. Item (4) is apparent from a comparison of Figs. 4(*a*) and 4(*b*) with Figs. 4(*c*) and 4(*d*). It is worthy of note that the injection of dye in the junction does not clearly define the line of demarcation between the two streams after they have joined. Notice that the line of separation is different in Figs. 4(*a*) and 4(*b*), whereas the photographs are of identical conditions, except as to the dye. The differences between the two lines must represent the region in which diffusion and turbulent mixing are taking place.

Application of the Results.—These results are not directly applicable to natural streams because of the restrictions placed upon the experimental pro-

cedure. A little reflection will show, however, that the problem is of minor importance if deep, slow-moving rivers are considered. The ratio of the velocity head to the depth is such a small number that the resulting ratio of depth above (y_a) to depth below (y_b) the junction will be close to unity. For example, consider a stream entering another at 45° and carrying 60% of the total flow. Suppose this tributary is 20 ft deep and is moving at 3 ft per sec.



(a) $\theta = 45°$; Dye in Channel 2 (b) $\theta = 45°$; Dye in Channel 1



(c) $\theta = 135°$; Dye in Channel 1 (d) $\theta = 135°$; Dye in Channel 2

Fig. 4.—Flow Patterns, Combining Flow

According to Fig. 3(a), the difference in depth between two points above and below the junction would be approximately 2%, or 0.40 ft.

The results show clearly that, owing to the negligible effect of friction, model studies of special stream intersections (if the problem were sufficiently important) would be successful. This procedure is recommended since, as stated, theoretical and experimental investigation of every conceivable type of intersection is impracticable.

Dividing Flow

In dividing flow the problem is somewhat different from the case of combining flow. A logical statement of this problem would be: A given stream branches into two separate channels. What will be the division of the flow? This will depend upon the backwater characteristics of the two branch channels and the dynamic conditions existing at the junction.

Analysis of the problem of dividing flow is considerably more difficult than that of combining flow, for the following reason: In the former case, it was

possible to assume that the depths in the tributary channels were equal immediately above the junction. No analogous assumption is permissible in the case of the dividing flow. The momentum equations are then complicated to the extent of containing an additional unknown term. In view of this fact, no attempt at rational analysis in terms of momentum was made, because assuming a relation among the three depths would be almost tantamount to assuming the solution to the problem. Furthermore, experience with combining flow has indicated that the theoretical analysis is in only fair agreement with the observed facts.

This part of the paper will be limited to the case in which the flow of water in Fig. 2 is reversed. The restrictions will be the same as for the combining flow, except that the flow is from channel 3 into channels 1 and 2.

It is evident that the relationship among the quantities flowing in each channel, and the depths therein, may be represented as some function of the seven following factors: The angle θ; the three depths, y_1, y_2, and y_3; any two of the three quantities, say Q_3 and Q_2; and the velocity with which the stream enters the junction. For any given angle θ, it is possible by dimensional considerations to reduce the remaining six variables to four. They are: $\frac{Q_2}{Q_3}$, $\frac{y_3}{y_2}$, $\frac{y_1}{y_2}$, and $\frac{(V_3)^2}{2\,g\,y_3}$ $(= k_3)$. It is proposed to present the experimental data for one type of intersection and then demonstrate how these data may be applied to a case that may occur in practice.

Experimental Results.—Fig. 5 gives the experimentally determined data for a channel division in which $\theta = 90°$. No comparison with theory has been made. Interpolated curves for values of k_3, other than test values, are shown as broken lines in Fig. 5. Fig. 6 shows the relationship that was found to exist among the three depths within the range of the values of k_3 considered. Figs. 7(*a*) and 7(*b*), which are of identical flow conditions, except as to the dye, show qualitatively the manner in which the flow divides.

Application of Results.—It has been stated previously that a practical use of a study in stream division would be to predict the division of flow when a given stream branches into two separate channels. It is assumed, of course, that the inflows to the branches are not controlled by gates but merely by the characteristics of the junction and the branches themselves. The following hypothetical case will be assumed to exist: A rectangular canal, 10 ft wide, divides at a 90° branch. The branch channels are also 10 ft wide and rectangular. They are of such slopes and roughness that their rating curves for sections just below the division point are as shown in Fig. 8. The problem is to predict how much water will be naturally diverted to each branch channel if 500 cu ft per sec are admitted to the main canal.

Determination of the division of flow involves the following steps:

(1) Assume a certain division, say $Q_2 = 250$ cu ft per sec and $Q_1 = 250$ cu ft per sec $\left(\frac{Q_2}{Q_3} = 0.5\right)$. This fixes the depths y_1 and y_2. The ratio of y_3 to y_2

may then be determined by means of Fig. 6. It is possible, by choosing various values of Q_2, to obtain a curve between $\frac{Q_2}{Q_3}$ and $\frac{y_3}{y_2}$. For the conditions assumed herein, this curve appears in Fig. 5 and is marked A.

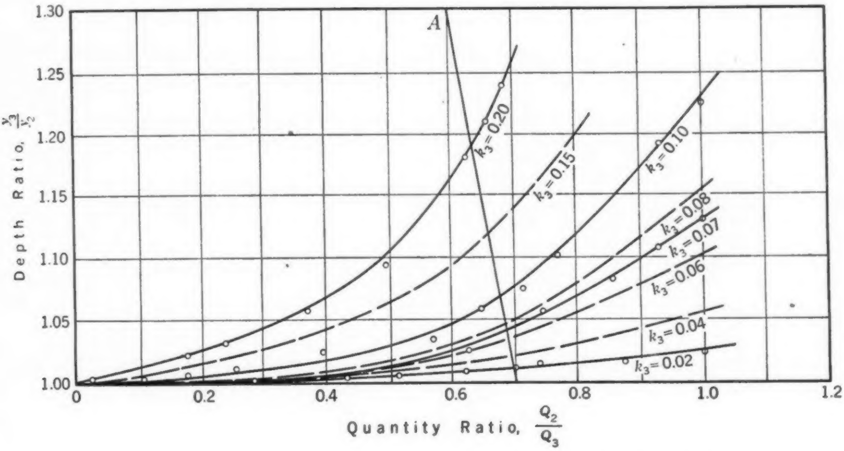


FIG. 5.—CHARACTERISTICS OF STREAM DIVISION IN WHICH $\theta = 90°$

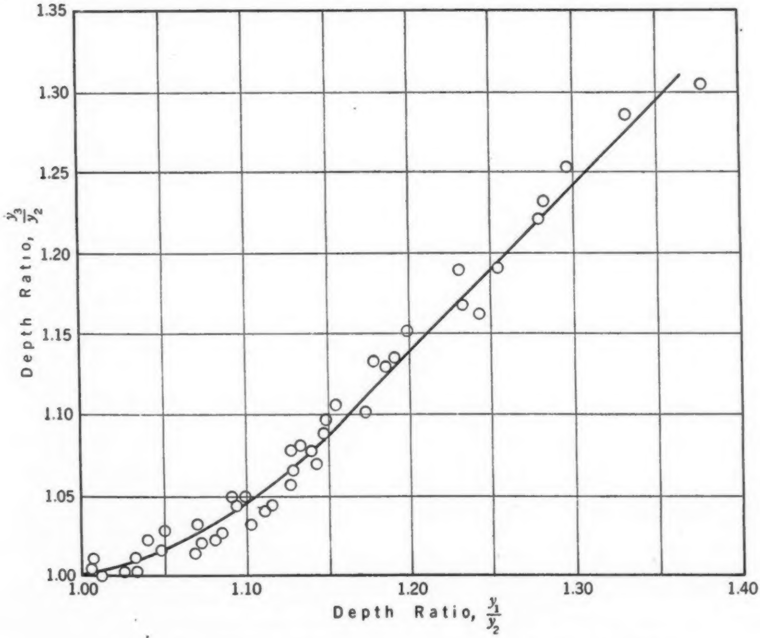
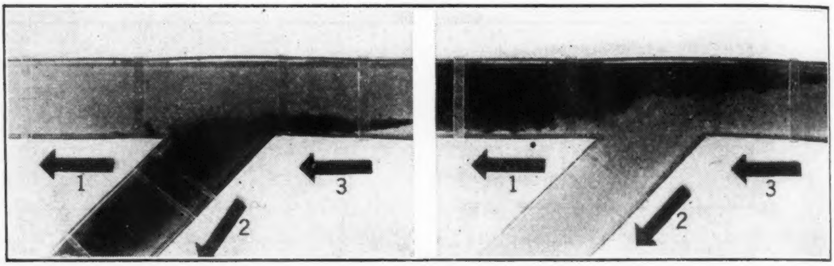


FIG. 6.—RELATIONSHIP BETWEEN DEPTHS IN A 90° STREAM DIVISION

(2) The locus of the points of intersection of curve A and the k_3-curves (Fig. 5) now gives all possible combinations of the variables. Still undetermined is the value of k_3 with which the flow of 500 cu ft per sec enters the



(*a*) Dye Entering Channel 1 (*b*) Dye Entering Channel 2

FIG. 7.—FLOW PATTERNS, DIVIDING FLOW; θ = 45°

junction. It is necessary to replot curve A in terms of the numerical values of the depth y_3 and the factor k_3. This is done as follows: Take the intersection of curve A and $k_3 = 0.10$; $\frac{y_3}{y_2} = 1.067$ and $\frac{Q_2}{Q_3} = 0.677$. Then $Q_2 = 338$ cu ft

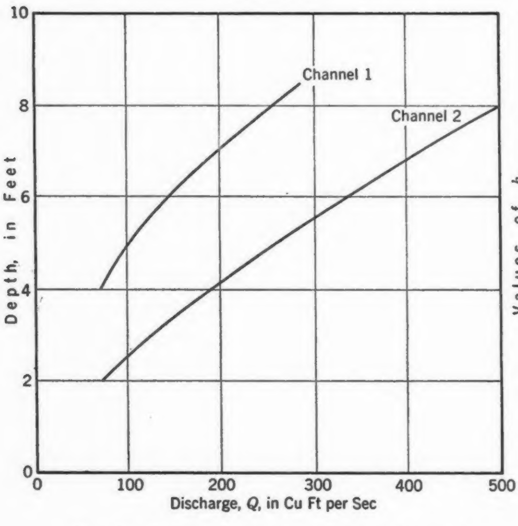


FIG. 8.—RATING CURVES FOR BRANCH CHANNELS

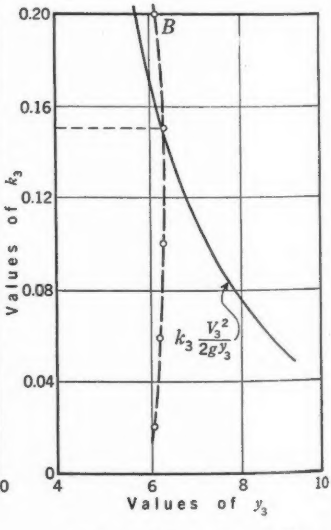


FIG. 9.—FACTOR k_3 AS A FUNCTION OF y_3

per sec and, from Fig. 8, $y_2 = 6.0$ ft and $y_3 = 1.067 \times 6 = 6.4$ ft. The point ($k_3 = 0.10$ and $y_3 = 6.4$) is plotted in Fig. 9. Other points are plotted in the same manner. The resulting curve is marked B in Fig. 9.

(3) The proper value of k_3 must satisfy not only curve B, but also the relationship $k_3 = \frac{(V_3)^2}{2\,g\,y_3}$. This is easily found by plotting $k_3 = \frac{(V_3)^2}{2\,g\,y_3}$ in Fig. 9 and obtaining the intersection with curve B. For these hypothetical conditions, k_3 is found to be 0.15 and $y_3 = 6.4$ ft.

(4) Entering Fig. 5 with $k_3 = 0.15$ at its intersection with curve A, the value of $\frac{Q_2}{Q_3}$ is seen to be 0.66. Then $Q_2 = 330$ cu ft per sec and $Q_1 = 500 - 330 = 170$ cu ft per sec.

Conclusion

The example given is admittedly a very special case, as such an ideal set of conditions could hardly be expected to occur in practice. It has only been included to indicate the data that would be necessary in order to solve a general problem of this type, as it is felt that rational analysis is not practical.

The research described in the foregoing may be considered by some to be incomplete in that no attempt was made to study the details of the mixing process in quantitative terms. Studies have since been undertaken, and are still continuing, at the University of California under somewhat simpler conditions; namely, the mixing of parallel streams, in which velocity and pressure gradients have been measured in the region of mixing.

Acknowledgment

The Committee of the Hydraulic Division on Hydraulic Research has had for one of its projects "The Phenomenon of Intersecting Streams." With funds allotted by The Engineering Foundation, the University of California conducted the investigation, with Morrough P. O'Brien, M. Am. Soc. C. E., as the cooperating member of the Committee.

The quantitative data from which this paper was prepared were taken from two sources: "Combining of Streams in Open Rectangular Channels,"[2] by F. C. Horowitz and John R. Morgan, Juniors, Am Soc. C. E., and "The Characteristics of Dividing Streams in Open Rectangular Channels,"[3] by Lt. Ivan C. Rumsey. The photographs were obtained from a thesis entitled "A Qualitative Study of the Intersections of Converging and Dividing Streams,"[2] by R. M. Bickerstaff, Jun. Am. Soc. C. E. R. L. Stoker was immediate supervisor of these studies.

APPENDIX

Notation

The following letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Hydraulics,[4] prepared by a Committee of

[2] Thesis submitted to the University of California in partial fulfilment of the requirements for the degree of Bachelor of Science in Engineering.

[3] Thesis submitted to the University of California in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

[4] ASA-Z10.2—1942.

the American Standards Association, with Society representation, and approved by the Association in 1942 (numeral subscripts refer to appropriate channel numbers, channel 3 being the main stream and channel 2 the channel entering or leaving the straight axis of flow):

b = width of rectangular channel section;
F = net force acting on a fluid system: F_{1-3} = net force acting in the direction 1 to 3;
g = acceleration of gravity;
k = a factor $= \frac{V^2}{2\,g\,y}$;
M = momentum of a system: ΔM_{1-3} = the rate at which momentum is changed in the direction 1 to 3;
n = ratio symbol: $n_q = \frac{Q_2}{Q_3}$ and $n_y = \frac{y_a}{y_b}$;
Q = rate of flow: Q_2 is the flow from or to the side tributary or branch channel; and $Q_3 = Q_1 + Q_2$ (see Fig. 2);
U = an unknown reaction in Eq. 1;
V = average velocity across a given cross section;
y = depth of flow: y_a = depth above the junction and y_b = depth below the junction;
γ = force per unit weight; specific weight of water; and
θ = angle between merging or diverging channels.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

RELATION OF UNDISTURBED SAMPLING TO LABORATORY TESTING

By P. C. Rutledge,[1] Assoc. M. Am. Soc. C. E.

Synopsis

Techniques and equipment for undisturbed sampling of soil, laboratory testing of such samples, the mechanics of applying the results of laboratory tests to engineering soils problems, and field observations of the effectiveness of the solution methods through the behavior of full-scale structures have frequently been considered as separate entities. They should not be so considered because each is a series of links in a chain which must reach an anchor or end result to achieve its purpose. Undisturbed sampling and laboratory testing have as their prime objective the prediction of how natural soil deposits will behave under proposed loadings. The purpose of this paper is to explore one link in this section of the chain—namely the extent to which laboratory test results approximate the physical properties of soil in natural deposits.

Since successful undisturbed sampling operations have been largely confined to plastic cohesive soils, this paper is limited to analyses of the effects of sample disturbance on the results of tests on natural clays. The analyses are based on consolidation, direct shear, unconfined compression, and triaxial compression tests on specimens ranging in disturbance from the least disturbed samples obtained to samples that were completely remolded.

Sampling of Plastic Cohesive Soils

During the early years of soil mechanics all laboratory tests were performed on soil specimens remolded from small-diameter samples taken in borings. If larger samples were taken by hand in pits, they were also remolded before test to maintain a uniformity in laboratory technique. In 1932, Arthur Casagrande, Assoc. M. Am. Soc. C. E. (1),[2] discovered that natural clays have a complex

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **April 1, 1943.**

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[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix).

intergranular structure that is destroyed by remolding and cannot subsequently be duplicated in the laboratory. Professor Casagrande's tests showed that there is a marked difference in the physical properties of undisturbed clay as compared with the same material remolded. The importance of determining physical properties from tests on soil in its undisturbed condition was at once apparent and undisturbed sampling came into being.

Most of the identification and classification tests used today have survived from the earlier days of soil studies, and their performance requires preliminary remolding of the soil. Therefore, they need not be considered in relation to undisturbed sampling. Laboratory testing of undisturbed samples is confined almost exclusively to those tests which measure physical properties. The most important of these tests are the consolidation tests, which measure the volume change of soil under load, and the strength tests, which measure the change in shape of soil under load and its maximum resistance to compressive or shearing stresses. The results of consolidation tests are used to analyze and predict settlements of structures. The important strength tests applied to soils are the unconfined and the triaxial compression tests and the direct shear test. Their results are used in the analysis of all problems of stability of soil masses, pressures on walls and tunnels, and ultimate bearing capacities of foundations.

In the best undisturbed samples obtainable, which are large cubical chunks of soil carefully cut out by hand in excavations, some disturbance of natural conditions is inevitable, due only to the removal of pressures of overlying and surrounding soil and their replacement by internal capillary tension in the water in the soil voids. In boring samples the degree of additional disturbance varies widely with the sampling equipment and techniques and the type of soil being sampled. Consequently, one cannot assume that the structure and physical properties of the undisturbed samples tested in the laboratory are those of the soil in the ground.

What are the effects of slight sample disturbance on laboratory testing? If samples are taken with reasonable care, using the more modern types of sampling equipment, the disturbance in plastic soils is not visually apparent. It has little or no effect on the ease of preparation of specimens for laboratory tests or on the performance of the tests. The effects of slight sample disturbance become evident only in the analysis and use of test results.

The established fact that laboratory test results are affected in some degree by inevitable sample disturbance is a challenge to the validity of laboratory soil testing and the application of test results to the solution of foundation problems. Therefore, this paper has the following three purposes:

(1) To analyze the effects of sample disturbance on the significant results of laboratory tests;
(2) To determine to what extent laboratory test results can be used in the solution of foundation and earthwork problems; and
(3) To find means for extrapolation from laboratory test results to the corresponding behavior of soil in nature.

These purposes can be fulfilled only by determining, or approximating with reasonable accuracy, the true initial conditions of the soil in the ground and its

subsequent changes in volume and shape under changes in stress. Only three methods are now available for investigating the relation between actual soil behavior and laboratory test results. These are: (*a*) Comparison of test results with field performance of full-scale structures, (*b*) direct reasoning based on the evidence in the test results, and (*c*) reasoning based on hypotheses for soil action which are supported by indirect field or test evidence.

Method (*a*) is greatly to be preferred, but only in a few cases are sufficient field observation data available for its use. As a single example, records of settlements due to compression of strata of soft clay underlying structures have been published and, in some cases, the settlements have been correlated with computations based on laboratory tests (2) (3) (4) (5) (6). The observations and correlations are sufficient to substantiate, in a general way, the methods of computing settlements taught by Karl Terzaghi, M. Am. Soc. C. E., for cases of heavy structures founded on or above reasonably uniform strata of very compressible clay or organic soils. The reported deviations of the admittedly approximate theory from observations are of the same order of magnitude as the deviations of stresses computed by conventional structural theory from published observed stresses in steel and concrete structures. The observational data are not sufficient to validate theoretical solutions for all cases of settlements or to confirm the inferences from laboratory tests in regard to the effects of sample disturbance on soils other than very soft clays.

Methods (*b*) and (*c*), which are not based on direct field observations, cannot establish the validity of applications of laboratory test results to field problems. Their function is to show whether the evidence now available supports or discredits the use of laboratory tests on undisturbed soil samples.

Sampling methods and techniques for laboratory testing of undisturbed samples have been applied successfully only to plastic cohesive soils. Therefore this paper will be limited to the principal tests for the physical properties of such soils; namely, the consolidation, direct shear, simple compression, and triaxial compression tests.

Sample Disturbance and Consolidation Test Results

The consolidation test is the laboratory means for measuring the relations between vertical stress, volume change, and rate of volume change of soil. The apparatus has been described in detail elsewhere (7). The test specimen is confined laterally by a rigid brass ring and is loaded vertically through porous stones. The load is usually applied in increments. For each load increment the sample changes volume slowly, and if this volume, or the void ratio of the sample, is plotted against the vertical stress after it has become substantially constant for each load increment, curves of the type shown in Fig. 1 are obtained. Fig. 1(*a*) is used in this paper because the semilogarithmic plot accentuates certain features of the test results. The void-ratio scale can be transformed linearly into a scale of volume or of volumetric strain if desired.

Fig. 2 shows an idealized test curve of void ratio plotted against vertical stress on a logarithmic scale. Over a substantial portion of the plot, the curve is a straight line for most natural clays with initial void ratios less than 1.5. The principal deviations from the straight line are found during the initial

loading of the sample and at points where the load is reduced. As a preliminary hypothesis one may reason by extrapolation that, as the soil deposit was formed, it was gradually compressed by a growing weight of overburden and that the void-ratio, pressure relationship followed the same straight line prolonged, as

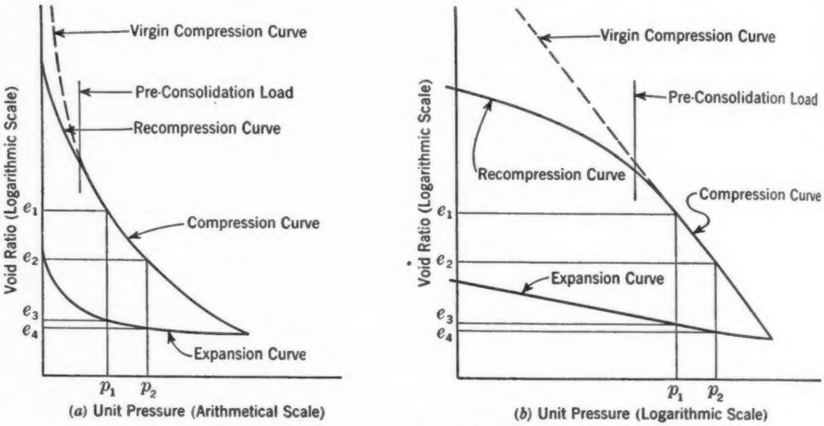


FIG. 1.—COMPRESSION DIAGRAMS

shown in the dash-line portion of curve 2, Fig. 2. Hence the straight-line portion of the curve is called the virgin compression curve. The formula for this curve is

$$e_x = e_0 - C_c \log \frac{p_x}{p_0} \qquad (1)$$

in which C_c is a compression index and e_0 and p_0 are the void ratio and unit pressure, respectively, at an arbitrary point on the straight line, and e_x and p_x are the corresponding values at any other point in the curve. When the sample was removed from the ground it expanded with the release of the pressure of the surrounding and overlying soil (curve 3, Fig. 2) and was again compressed in the laboratory test (curve 1). Following this reasoning, one can assume that the expansion during and after sampling, and the recompression during the initial part of the consolidation test, are identical to expansion and recompression which can be produced in the laboratory test, as shown in curves 5 and 4, Fig. 2. It is then reasonable to assume that the relation between the maximum vertical stress exerted on the soil in nature will be related to the initial laboratory compression curve (1) in the same way that some larger laboratory stress is related to its corresponding laboratory recompression curve (4). This maximum natural vertical stress is called the "preconsolidation load" (8) and can be determined approximately from the shape of the initial compression curve.

The effects of slight sample disturbance can be investigated by carrying the disturbance to an extreme—in other words, by remolding the test specimens. Figs. 3, 4, and 5 show the results of four sets of tests in each of which one test specimen, after being tested as undisturbed sample, was completely remolded with the addition of water to make its water content and void ratio as nearly

equal as possible to the condition of the sample before the test. In this condition of being completely disturbed, or remolded, at the natural water content, these specimens were retested. The curves in Figs. 3 and 4 show the results for an organic clay and two glacial clays. The entire curves for the remolded

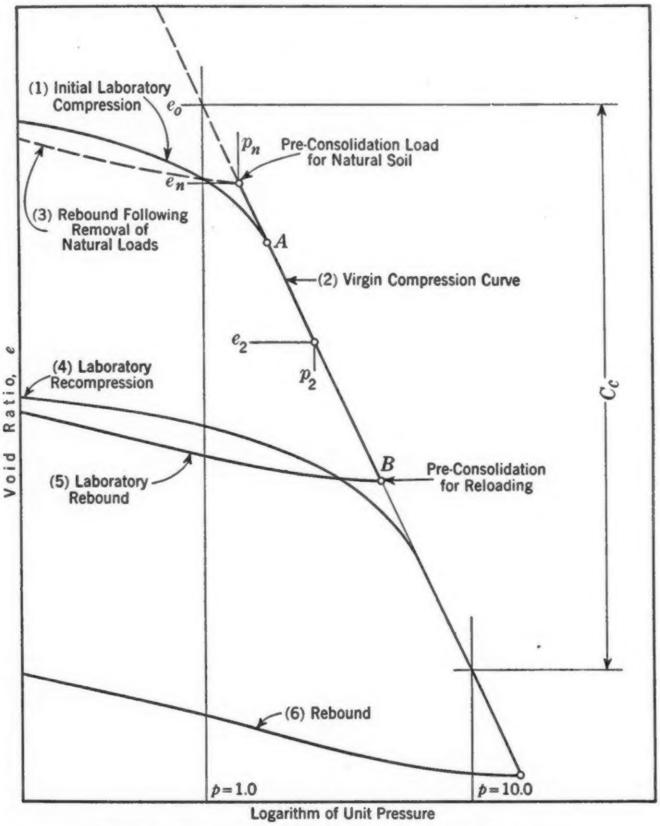


FIG. 2.—TERMINOLOGY FOR THE COMPRESSION DIAGRAM

tests are displaced downward on the scale of void ratio. Thus, the void ratios for any particular vertical stress are much smaller than the void ratios corresponding to the same stress on the undisturbed specimens. Straight-line compression curves are again obtained but, in each case, they have a flatter slope than the compression curves of natural soils obtained from the tests on the undisturbed specimens in the laboratory. Also the initial parts of the compression curves slope very gradually into the straight-line portions and no definite preconsolidation loads are indicated. Prolongations of the laboratory virgin compression curves for the undisturbed samples and the straight-line compression curves for the remolded tests tend to intersect. Thus, volume change under some load would tend to remold the soil completely.

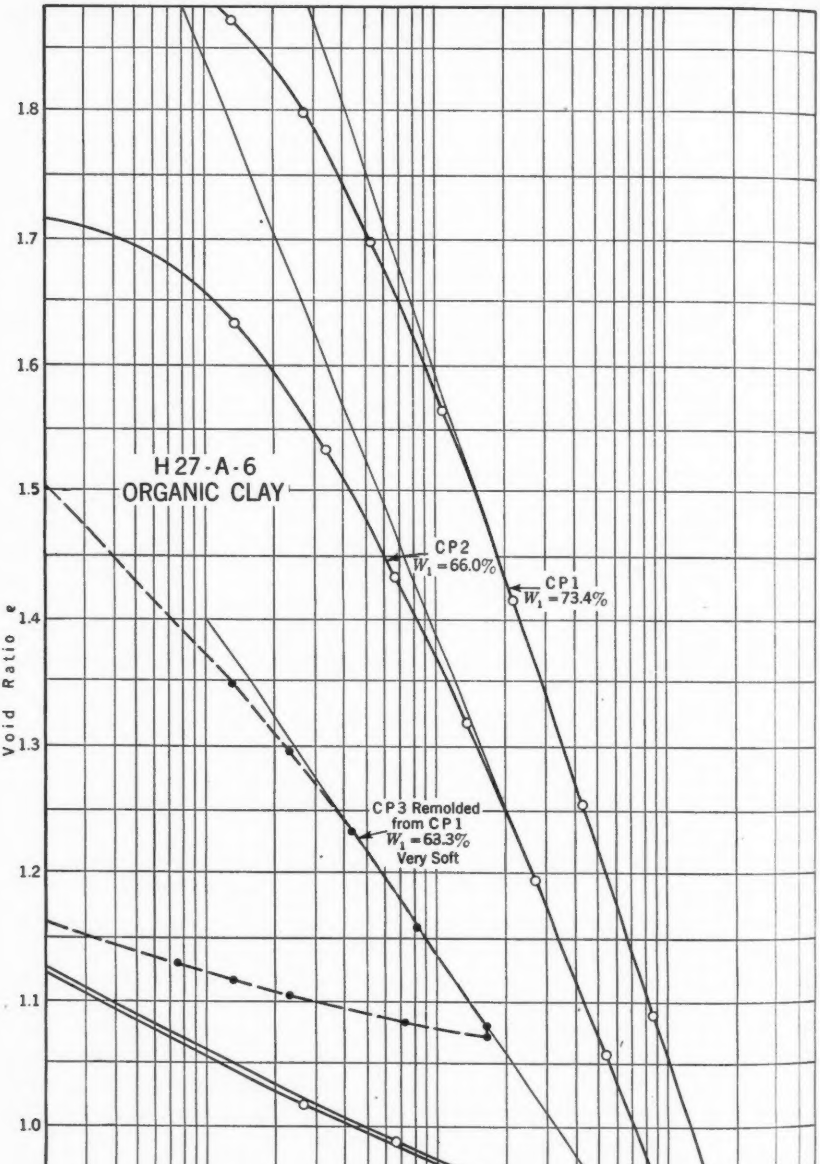


FIG. 3.—EFFECT OF REMOLDING ON THE COMPRESSION CHARACTERISTICS OF NATURAL ORGANIC CLAY (CLASS H27–A–6)

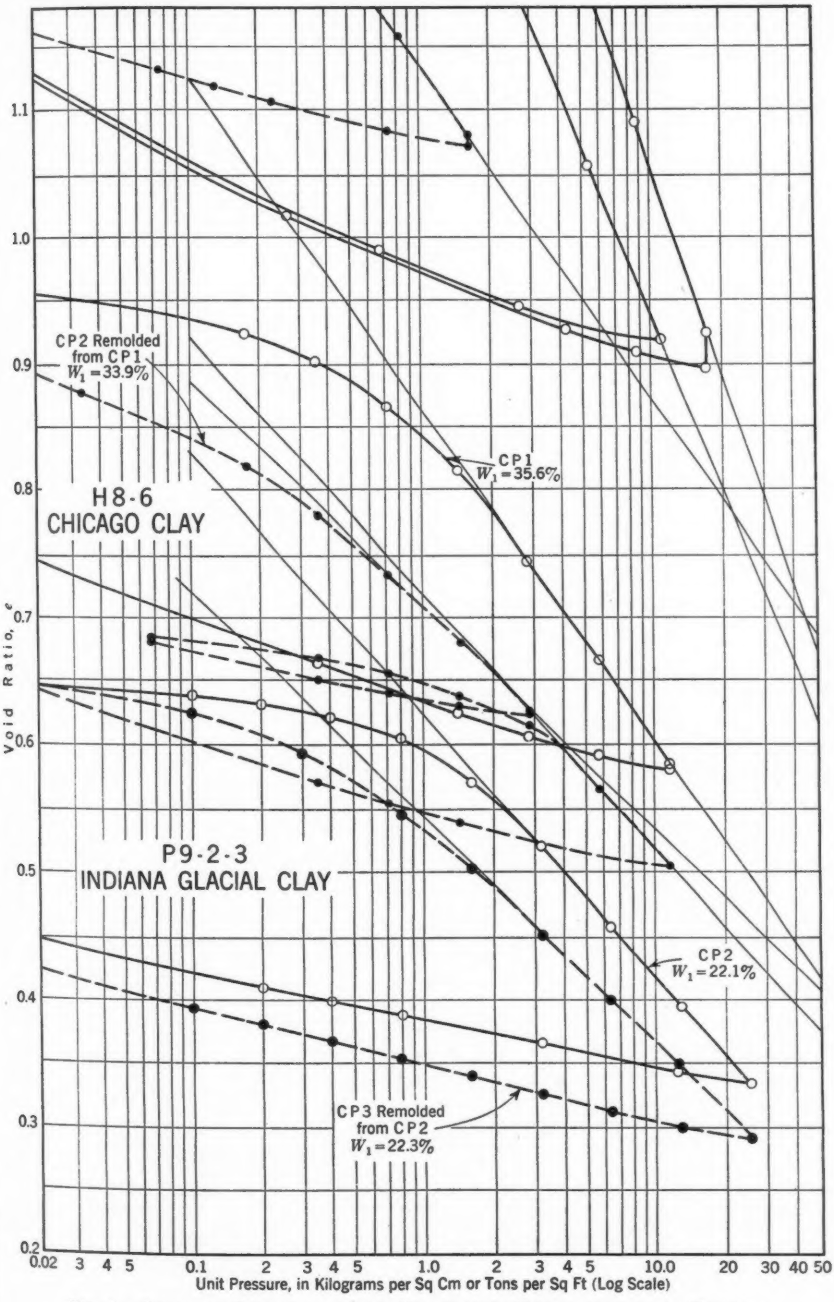


FIG. 4.—EFFECT OF REMOLDING IN THE COMPRESSION CHARACTERISTICS OF NATURAL CHICAGO CLAY (CLASS H8-6)

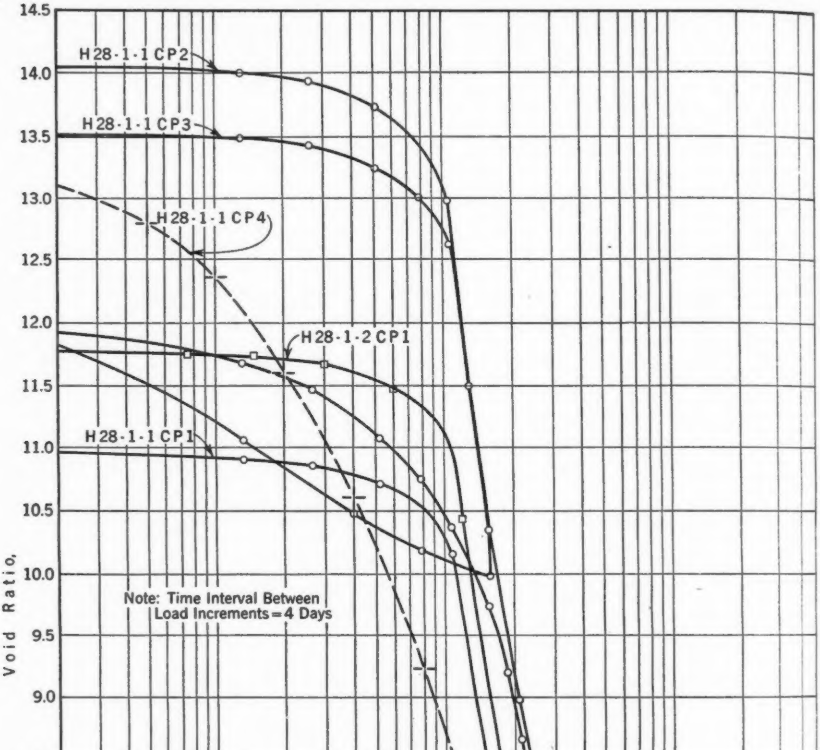
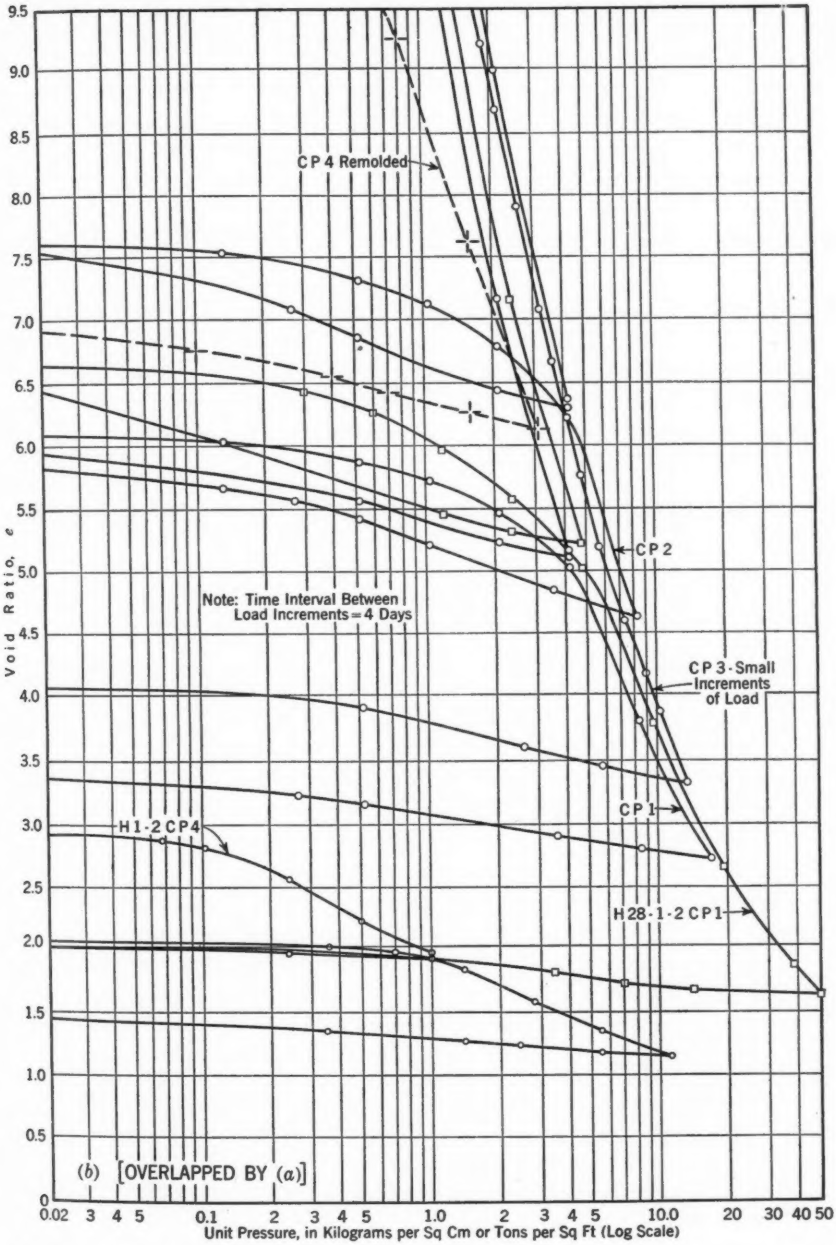


FIG. 5.—EFFECT OF REMOLDING ON THE COMPRESSION



Characteristics of Mexico City Clays

Fig. 5 shows the results of similar tests on a highly compressible volcanic clay from Mexico City, Mexico. The sample from which these test specimens were taken was a 12-in. cube carefully cut out by hand and shipped to the laboratory with great care. The soil was disturbed very little and, as a result, the initial compression curves for the tests on undisturbed samples break very sharply into the laboratory virgin compression curves. The tests show that the preconsolidation load should be almost exactly 1.0 ton per sq ft. The actual weight overlying the specimen was 0.97 ton per sq ft. The nature of this material is extreme in that, fully consolidated under 1.0 ton per sq ft, it had a void ratio of 14.0. In other words, only one part in fifteen of its volume was solid mineral matter. As the curves in Fig. 5 show, this material underwent tremendous volume changes under load and the resulting laboratory virgin compression curves are not straight lines. Compression curves of tests on natural soils which are slightly concave upward have also resulted from tests on other clays with initial void ratios greater than 1.5. Such curves probably are an indication of minimum sample disturbance. The compression curve for the test on a specimen remolded at its natural water content is a straight line as far as the test was carried. It falls below the curves for undisturbed tests and gives every indication of meeting tangentially and continuing with these curves.

The four sets of test results shown in Figs. 3, 4, and 5 are typical of many others and cover an extreme range in plastic soils. Remolding changes these compression curves for undisturbed samples in the following ways:

(1) It decreases the void ratio at which the soil will carry any given vertical stress;

(2) It obscures the previous stress history of the soil and its preconsolidation load; and

(3) The straight-line portion of the remolded compression curve is displaced downward from the laboratory virgin compression curve, and its slope, or the rate of decrease in void ratio with increasing stress, is less.

These results show definitely the effects of sample disturbance over the range from the best undisturbed samples which have been obtained for laboratory test to complete remolding. Any intermediate degree of disturbance must result in a compression curve which falls between the curves that would be obtained for the same soil in these two known limiting conditions. On this basis compression curves are plotted in Fig. 6 for (1) a good undisturbed specimen, (2) the same soil slightly disturbed, and (3) the same soil remolded. Curve (2), Fig. 6, is typical of actual test results on many specimens from samples taken in borings. In relation to curve (1) the effect of the previous stress history of the soil is less marked; but it is possible to estimate the preconsolidation load with decreased accuracy.

In Fig. 6, point B on curve (2) for a slightly disturbed specimen represents the preconsolidation load for the given soil. The corresponding void ratio is considerably less than the initial void ratio of the sample at point C. Without referring to curve (1), two possibilities are apparent: (*a*) The sample represented by curve (2) has expanded during and after sampling and been recompressed

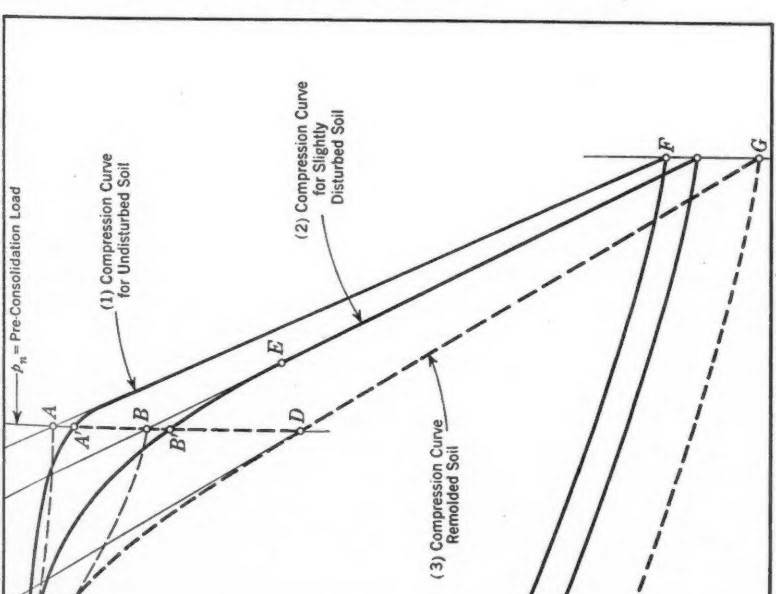


FIG. 6.—EFFECT OF REMOLDING AND SLIGHT DISTURBANCE ON THE SHAPE OF THE COMPRESSION CURVE

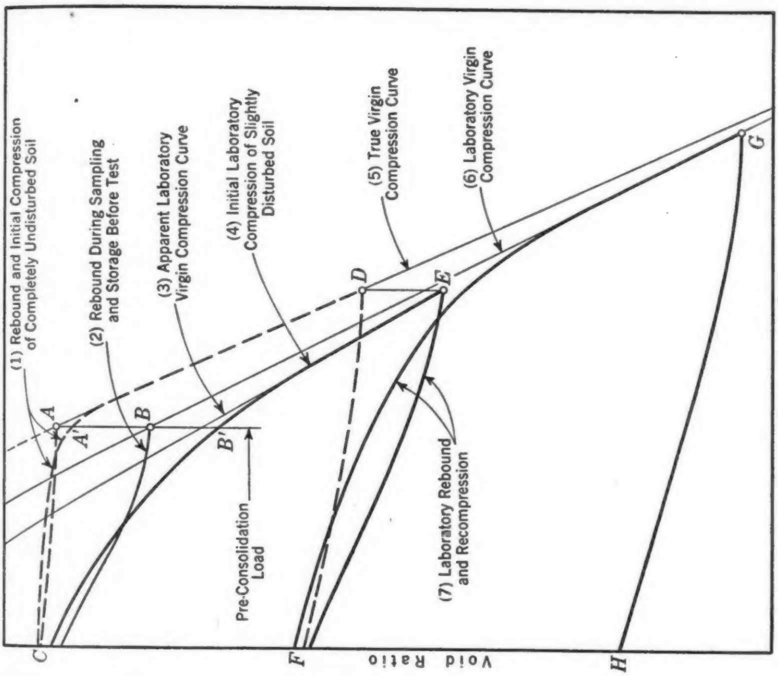


FIG. 7.—EFFECT OF DISTURBANCE DURING SAMPLING ON THE DETERMINATION OF THE PRECONSOLIDATION LOAD

in the laboratory test; or (*b*) point B is not representative of both the natural void ratio and preconsolidation load in the ground. Curve (1), Fig. 6, which is typical of test results that have been obtained for specimens from samples carefully taken by hand, indicates that point A represents more nearly the actual conditions of the soil in nature. Messrs. Casagrande (8) and M. J. Hvorslev, Assoc. M. Am. Soc. C. E. (9), have stated that saturated plastic cohesive soils probably swell very little during carefully conducted sampling operations.

Fig. 7 is a reconstruction of what may have happened to a slightly disturbed sample. Assume initially that the preconsolidation load is equal to the natural overburden pressure and that its relation with the natural void ratio of the soil in the ground is shown by point A. During the sampling operations the structure of the soil is weakened and it tends to compress under the preconsolidation stress to equilibrium at a new void ratio at point B. At the same time, the preconsolidation pressure is removed by the sampling operations and the sample is held at a constant volume by its small permeability which resists rapid outflow of water filling its voids and by the surface tension of the water at its free surfaces. The actual intensity of capillary pressure acting on the soil may be very small (it may be zero) because the reduction in strength due to disturbance removes the tendency of the mineral grain structure to expand. When a specimen from this sample is tested in the laboratory, it yields the heavy solid-line type of curve shown in Fig. 7. The preconsolidation load determined from this laboratory test falls on curve (6), Fig. 7, at point B. If point B can be determined with reasonable accuracy and if the initial void ratio of the test specimen (point C) is substantially that of the soil in nature, the intersection at point A of the preconsolidation load and the initial void ratio actually determines the condition of the soil in the ground.

Under these assumptions point A in Fig. 7 represents the natural conditions of soil with acceptable accuracy. The compression curve which will be followed actually by the undisturbed soil in the ground when it is loaded should then begin at point A. Beyond this point the investigator does not know its shape or its slope conclusively. He may assume that it is a straight line, parallel to the best laboratory virgin compression curve he can obtain. Laboratory investigations by the writer of many clays indicate that it may be a straight line slightly steeper than curve (6), Fig. 7. For practical purposes these two possibilities are identical and are shown by the broken line AD in Fig. 7.

In 1941, Professor Terzaghi presented a new hypothesis for the action of undisturbed clays which assumes a "rigid bond between the solid parts of the absorbed water films which surround the clay particles" (10). This hypothesis is based on, and explains some, field and test data not readily explained by existing conceptions. According to Professor Terzaghi, if the clay is loaded very slowly, as by the gradual deposition of overlying layers of soil over thousands of years, the "solid water" bond between particles is not broken and compression proceeds as shown by curve (2), Fig. 8. This very flat, natural compression curve has little or no relation to the results of laboratory tests on samples in which slight disturbance has broken the "solid water bond" and transformed the clay into a "lubricated" state. The possibility of its existence,

however, does not affect the validity of the preceding analysis of the results below the point A in Figs. 6 and 7 of comparatively rapid laboratory tests. That analysis is based on concrete test evidence and is independent of the way the soil arrived at the condition indicated by point A. On the other hand, extrapolation of curve (5), Fig. 7, above point A may not correspond to natural

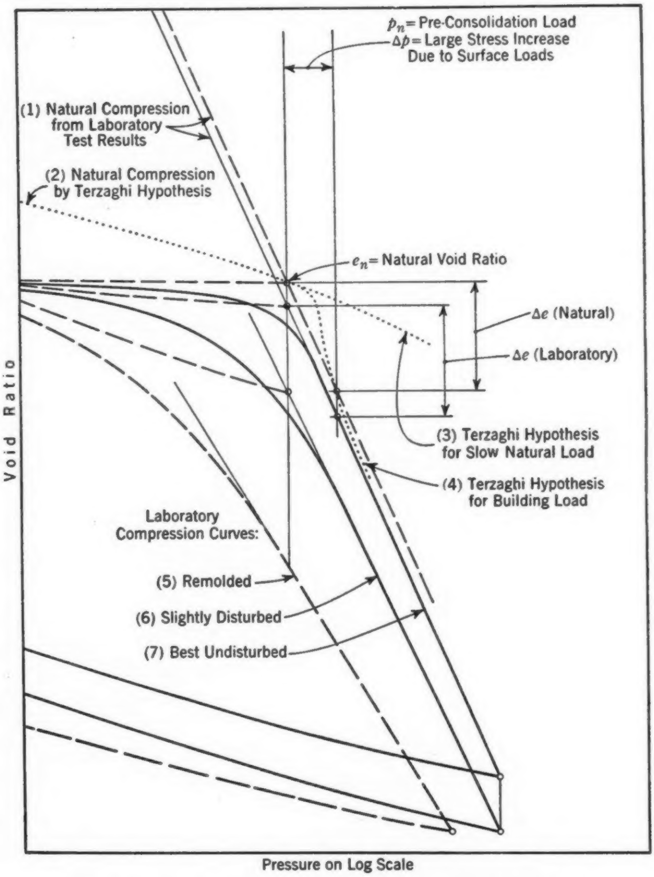


FIG. 8.—LABORATORY CURVES VERSUS COMPRESSION CURVES

phenomena. In a natural soil deposit loaded by a building, the clay may tend initially to follow some flat natural compression curve. Then, as the stress increase in the clay becomes larger, the bond might be broken and the clay might begin to compress as it does in the laboratory test. If this is the case, at stresses somewhat in excess of the preconsolidation load, the compression curve for the soil in the ground would break sharply downward toward the laboratory test curve. This is shown by the lower dotted curve (4) in Fig. 8. The tests on undisturbed specimens of Mexico City clay, shown in Fig. 5, re-

sulted in curves of this shape which break sharply and become almost vertical at a stress slightly in excess of the preconsolidation load. This is a phenomenon that has also been observed in a few tests on other clays with initial void ratios between 1.5 and 7.0.

The importance of these considerations is obvious. The engineer wishes to use the results of a consolidation test to predict the volume change in a natural undisturbed clay under an increase in stress caused by a building load, and consequently, the amount the building will settle. To do this, he must begin with the existing natural conditions and determine the actual relation between decrease in volume and increase in stress. The slight inevitable change in condition that occurs in the best undisturbed samples makes it inaccurate to apply the test results without modification. The following conclusions are a practical interpretation of the evidence now available:

(1) The preconsolidation load can be determined with reasonable accuracy from the results of consolidation tests on specimens from good undisturbed samples—for example, 8-in. to 12-in. cubical samples carefully cut out by hand in a test pit.

(2) Serious disturbance during sampling makes inaccurate, or prevents entirely, the determination of the preconsolidation load.

(3) If the Terzaghi hypothesis is accepted (and it seems to explain discrepancies between predicted and observed settlements under small stress increases as well as observed uniformities of water content over large depths of uniform clay strata) volume changes corresponding to changes in vertical stress in natural clay caused by foundation loads may be computed as follows:

(*a*) Volume changes caused by very small increases in stress will follow curve (3), Fig. 8, and may be neglected.

(*b*) Volume changes caused by larger increases in stress will break the rigid solidified water bond between clay particles. For all practical purposes the compression will then parallel the laboratory virgin compression curve (7) from the intersection with the preconsolidation load.

(*c*) The division point between these two possible actions has not been determined by field evidence. In present applications its selection is a matter of judgment, a division point at zero change in stress representing the present method of computation neglecting the Terzaghi hypothesis.

From these conclusions it is evident that a purely theoretical solution for settlement problems is not consistent with the character of the data derived from test results.

The effect of sample disturbance on test rates of volume change and prediction of rates of settlements of structures founded on clay soils has not been considered. There is little test evidence to show whether or not sample disturbance has any marked effect on rate of volume change except in so far as it affects the slopes of the compression curves. If the natural compression follows substantially the laboratory virgin compression curves, Professor Ter-

zaghi's original theory of consolidation (11) (12) probably is adequate and well within the accuracy of field determination of drainage conditions. If the natural compression occurs without breaking the "rigid clay bonds," the rate will be entirely different from the rates obtained from laboratory tests, but the magnitudes of the settlements probably will be small and the rates unimportant.

Strength Test Results

The strength tests are those tests which measure the resistance of soil specimens to change in shape and their ultimate strengths in shear, compression, or tension. The principal types of strength tests for soils are the direct shear, the simple or unconfined compression, and the triaxial compression tests. Within these three types many variations in test equipment and technique are found. They have been described in detail by many writers, including a symposium on shear testing published by the American Society for Testing Materials (13) and a recent comprehensive listing by B. K. Hough, Jr., Assoc. M. Am. Soc. C. E. (14).

The Direct Shear Test.—Direct shear testing of remolded clay has been studied exhaustively by Mr. Hvorslev (15); and L. Jurgenson (16) has studied direct shear tests on both undisturbed and remolded clays. These studies show that the shearing resistance of a clay depends on a number of factors, among which the previous stress history of the soil and its preconsolidation load are important. Consolidation test data have shown that sample disturbance modifies the effects of previous natural stresses in clays. Therefore, the shearing resistance must be affected in some degree. The direct shear-test data are inconclusive in regard to the magnitudes of effects of slight disturbance or remolding on the shearing strength of undisturbed clay. The only conclusion to be drawn from the existing data is that sample disturbance definitely affects the resistance of clay specimens to direct shear. The magnitude of the effect cannot be estimated with any assurance.

The Unconfined Compression Test

The unconfined compression test is easier to perform on undisturbed samples of plastic soil than the direct shear test and shows promise of becoming the more generally accepted of the two. It is performed on prismatic or cylindrical specimens in a manner almost identical to compression tests on concrete and other engineering materials. Precautions are taken to distribute the axial load uniformly to the ends of the specimen and the specimen is protected against evaporation. Longitudinal deformations are observed during the test. The results are plotted as longitudinal strain versus longitudinal stress acting on the average actual cross-sectional area for each load. The maximum longitudinal stress sustained by the sample in an unconfined compression test is called its "compressive strength." The slope of the initial portion of the stress-strain curve, which is frequently a straight line for the best undisturbed specimens, is called the "modulus of deformation" or the "modulus of elasticity" (17). Many natural clays are elastic in the sense that stress and strain are approximately proportional for continuously increasing compressive stress. They are not elastic in the sense that they will return to their original shape upon release

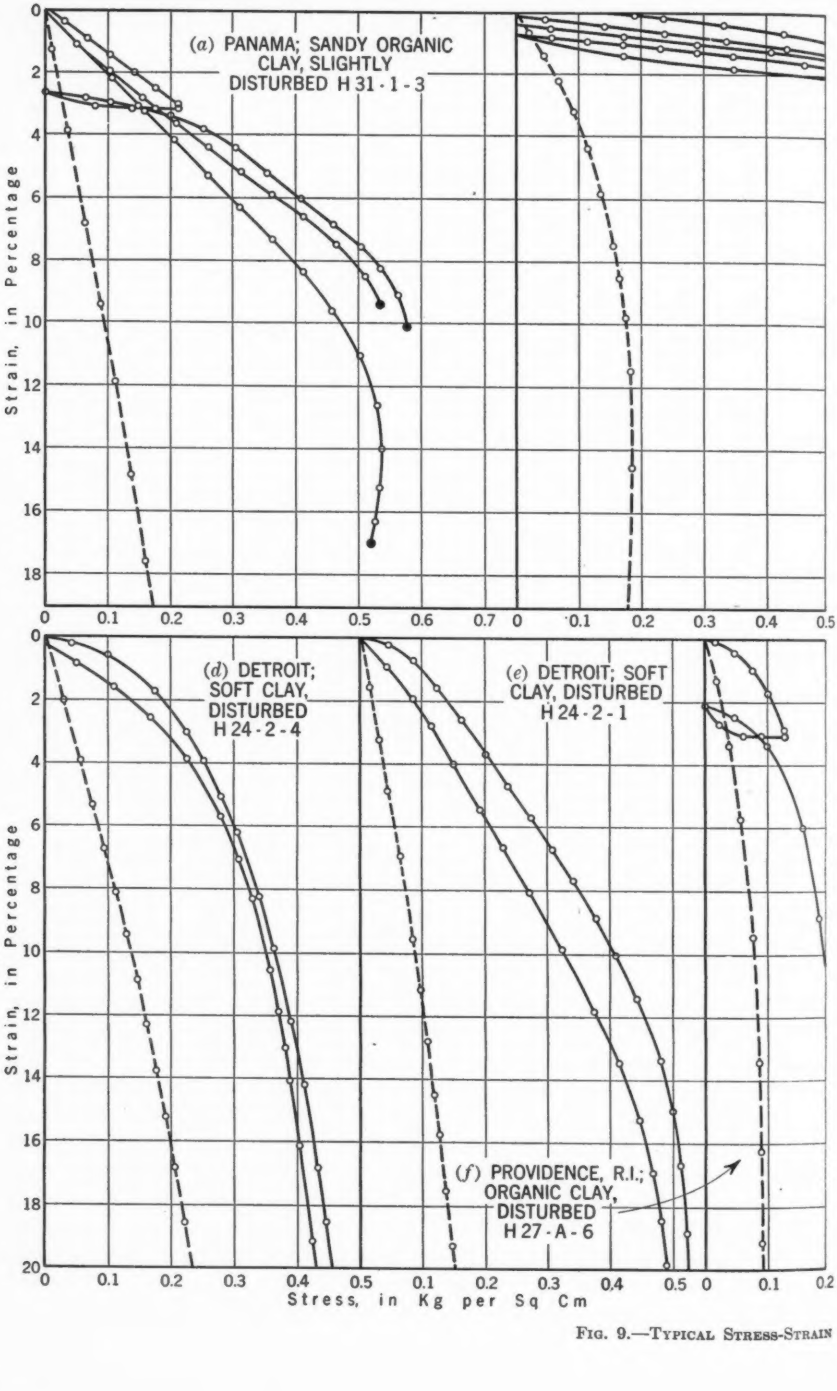
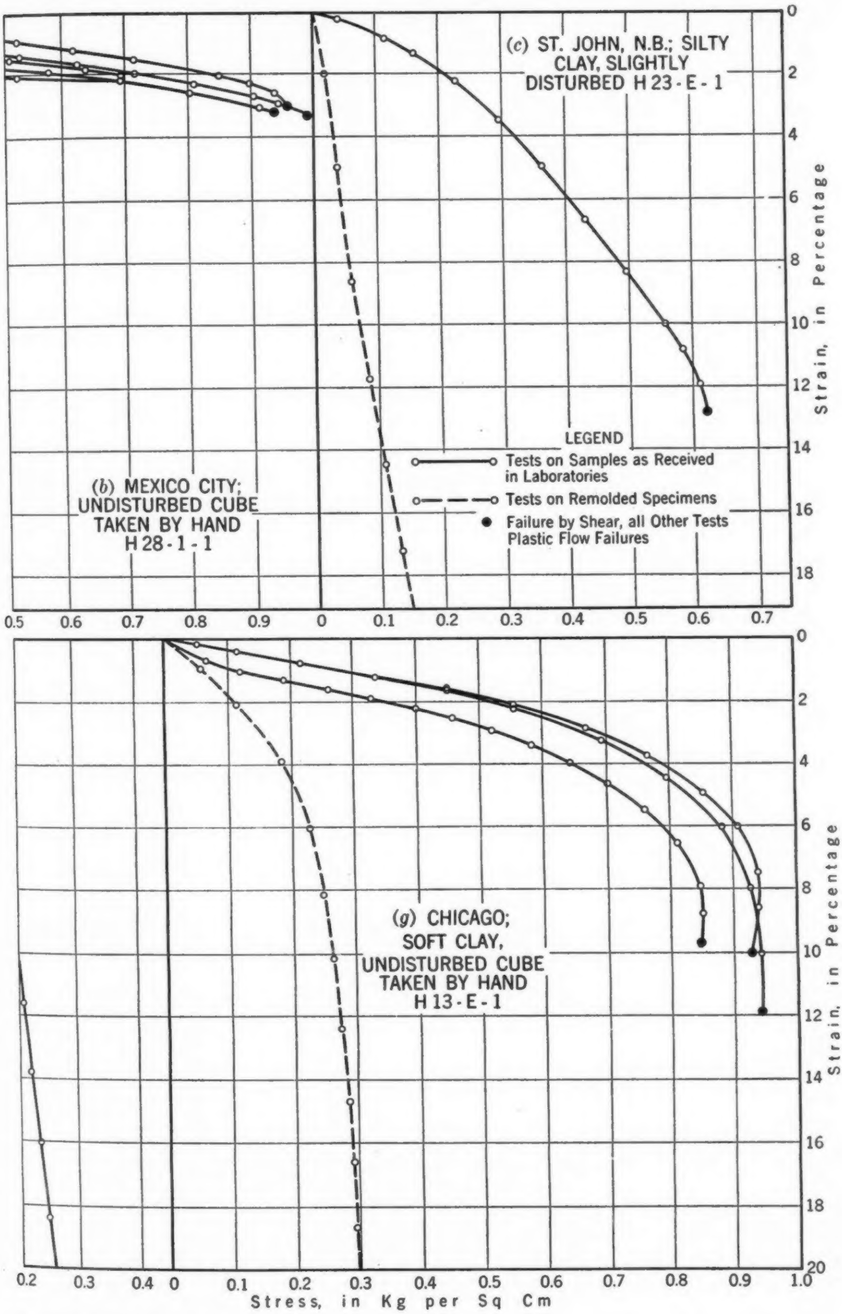


FIG. 9.—TYPICAL STRESS-STRAIN



CURVES FOR NATURAL SOILS

of stress. The term "modulus of elasticity" is used because it is the commonly accepted term for the coefficient of proportionality between stress and strain for specified conditions of loading, and because most field loadings of concern to engineers cause continuously increasing compressive stress in soil deposits (17). The use of this term should not imply that the modulus can be used in elastic theory without review of its limitations.

Fig. 9 shows a group of typical stress-strain curves for unconfined compression tests on natural clays. Specimens from each of these undisturbed samples were tested in the condition the samples were received. Then the undisturbed specimens were remolded to form one specimen for a test on the same soil in a completely disturbed condition. The difference between the results of tests on undisturbed and remolded specimens is striking. Specimens of Mexico City clay (Fig. 9(*b*)) were cut from the cubical sample described under consolidation tests and were little disturbed. The Chicago clay sample (Fig. 9(*g*)) was a similar cube cut out by hand in an excavation. The two sets of curves for these samples are representative of results of good tests on undisturbed samples. In the writer's experience tests on specimens from such samples invariably result in stress-strain curves which are straight lines to 30% or 40% of the

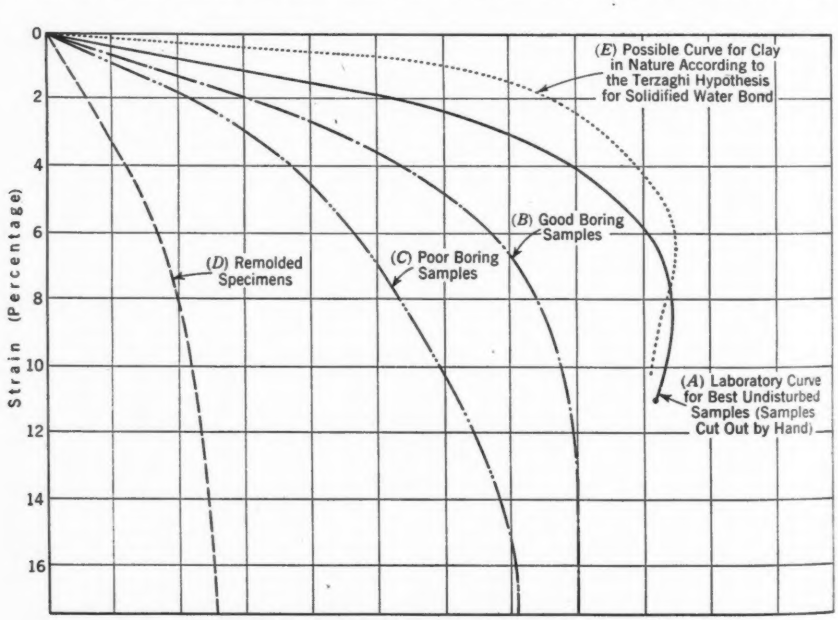


FIG. 10.—EFFECTS OF SAMPLE DISTURBANCE ON STRESS-STRAIN CURVES FROM UNCONFIRMED COMPRESSION TESTS

maximum stress, whether the soil fails by rupture at 2% strain or by plastic flow at 20% strain. In contrast to these curves, the curves for the St. John, New Brunswick (Fig. 9(*c*)), Detroit, Mich. (Figs. 9(*d*) and 9(*e*)), and Providence, R. I. (Fig. 9(*f*)), clays are typical for tests on specimens disturbed during

sampling. Curvature within the first 20% of the maximum load on a stress-strain curve is almost positive evidence of sample disturbance. Two series of tests by the writer on undisturbed specimens have shown that these results are not measurably affected by rates of application of strain varying between 1% per min and 1% per hr.

A specific study of the effects of disturbance during sampling on the results of unconfined compression tests was undertaken in connection with the soil investigations for the Chicago subway. The results were reported by R. B. Peck, Jun. Am. Soc. C. E. (18), in 1940. Mr. Peck has presented stress-strain curves which are remarkably similar to curves for approximately corresponding degrees of disturbance in Fig. 9. He plotted the test compressive strengths of remolded and of thin-walled seamless-steel tube samples against corresponding test compressive strengths for specimens from samples carefully cut out by hand, and found that compressive strengths varied between 65% and 75% of the "undisturbed" strength and the remolded strengths were 22% to 45% of the "undisturbed." The relative effects of sample disturbance on the initial slopes of the curves were much greater (18*a*).

As a result of the study of these test curves, and many others, with the same general characteristics, it is possible to formulate an hypothesis for the effect of sample disturbance on the results of unconfined compression tests. In Fig. 10, curve (A) represents typical results of laboratory tests on the best undisturbed specimens—for example, those cut from large cubical samples taken by hand in test pits or excavations; curve (B) represents typical results for tests on samples taken in bore holes by the most modern types of sampler; and curve (C), tests on disturbed samples taken by undisturbed sampling

TABLE 1.—EFFECT OF REMOLDING ON COMPRESSIVE STRENGTHS AND MODULI OF ELASTICITY OF CLAYS

(Kilograms per Square Centimeter)

Source	Material		Compressive Strengths			"Moduli of Elasticity"		
	Type of clay	Tested by[a]	Undisturbed	Remolded	Ratio, remolded/undisturbed	Undisturbed	Remolded	Ratio, remolded/undisturbed
Chicago	Glacial	Peck (18)	0.50	0.12	0.24	24	0.8	0.03
Chicago	Glacial	Rutledge[b]	0.94	0.30	0.32	28	5.7	0.20
Chicago	Glacial	Rutledge (19)	2.20	0.76	0.34	130	8.0	0.06
Chicago	Glacial	Rutledge (19)	4.1	2.1	0.51	295	44	0.15
Boston	Marine	Casagrande (1)	2.3	0.3	0.13	102	18	0.18
Laurentian	Marine	Casagrande (1)	6.1	0.4	0.07	740	28	0.04
Mexico City	Volcanic	Rutledge[b]	0.96	0.18	0.19	45	3.5	0.08

[a] Numerals in parentheses refer to the corresponding items in the Bibliography.
[b] Data taken from Fig. 9.

methods in bore holes. Tests on these same specimens, remolded at their natural water contents, would yield results shown approximately by curve (D), Fig. 10. Test measurements of the reductions in compressive strengths

and in "moduli of elasticity" from curve (A) to curve (D) (Fig. 10) are shown in Table 1.

According to Professor Terzaghi's hypothesis (10) for the "solid" versus "lubricated" states of natural clays, the rigid cementing action of the solidified water bond between clay particles may cause the soil to offer much greater resistance to deformation in its natural condition in the ground than can be obtained in a laboratory test. Thus the true initial slope of the stress-strain curve may be much flatter and the "modulus of elasticity" much greater than the best test specimens indicate. Small deformations, however, will destroy this bond as effectively as small deformations during sampling and, beyond a strain of 1% or 2%, the true stress-strain curve for the soil should be essentially parallel to, or should approach, the curve obtained from a laboratory test on the least disturbed type of specimen. On the basis of this hypothesis the true stress-strain curve for natural clay may have a shape similar to curve (E), Fig. 10. The compressive strength will practically be the same as for curve (A) if the strain at the maximum strength is greater than 2%. This conclusion has been verified partly by correlation of unconfined compressive strengths of samples carefully cut out by hand with field observations of lateral pressures exerted on the sheeting and bracing of open-cut sections of the Chicago subway (20).

These observations can be summarized by the following conclusions in regard to the effect of sample disturbance on the results of unconfined compression tests:

(1) Serious sample disturbance is indicated if, (*a*) the stress-strain curve is not a straight line for the first 30% of the compressive strength, or (*b*) the stress-strain curve falls close to a stress-strain curve for the same specimen remolded.

(2) Compressive strengths determined from tests on specimens cut from large cubes taken by hand are probably equal, for practical purposes, to the compressive strengths of completely undisturbed soils.

(3) Test compressive strengths of samples which are slightly disturbed during sampling may be considerably less than the compressive strengths under conclusion (2).

(4) The "modulus of elasticity" of a clay in nature will not be less than that obtained from the tests described under conclusion (2), but it may be greater by an amount which cannot be predicted from the results of laboratory tests.

The Triaxial Compression Test

In the triaxial compression test a cylindrical soil specimen, enclosed in a thin rubber membrane, is placed under a constant hydrostatic pressure and then tested in compression. A schematic diagram of typical equipment for this test is shown in Fig. 11. The additional compressive load in the direction of the axis of the specimen is applied by a piston which slides freely in an opening in the top of the pressure chamber. The axial compressive stress in excess of the constant hydrostatic pressure is called "deviator stress." The test results are plotted as a curve of axial strain versus deviator stress. Volumetric strains are measured by the quantity of water flowing into or out of saturated specimens and are also plotted against the deviator stress.

Triaxial compression tests on plastic, cohesive soils are divided into quick tests, consolidated quick tests, and consolidated slow tests. All have the advantage that the lateral forces of the hydrostatic pressure simulate the confining effect of surrounding soil in nature. Thus lateral expansion of the

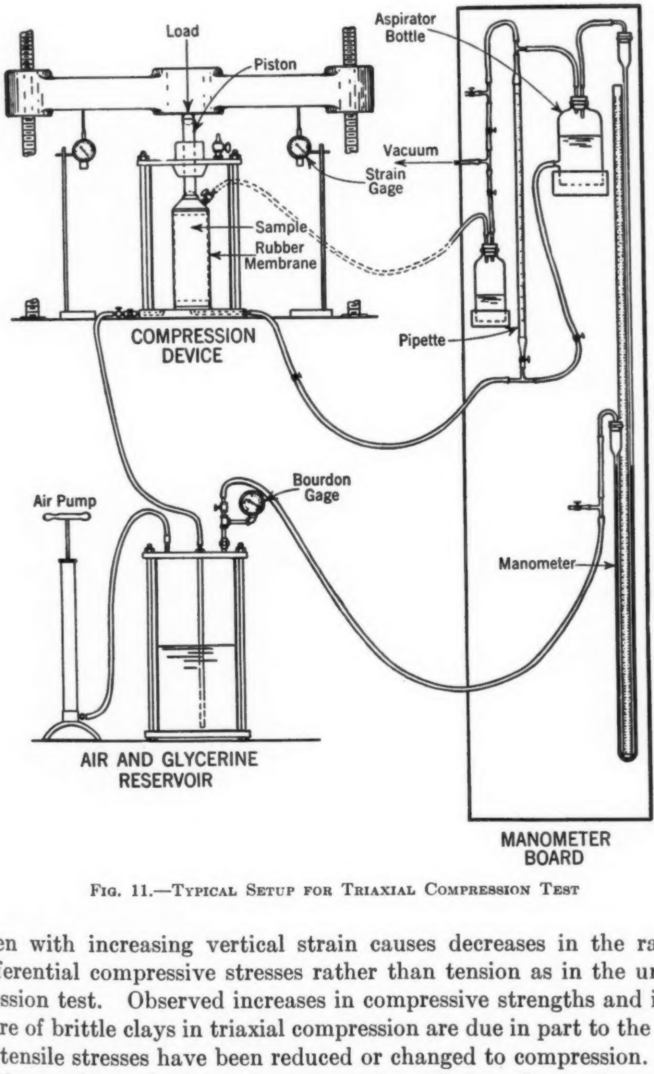


Fig. 11.—Typical Setup for Triaxial Compression Test

specimen with increasing vertical strain causes decreases in the radial and circumferential compressive stresses rather than tension as in the unconfined compression test. Observed increases in compressive strengths and in strains at failure of brittle clays in triaxial compression are due in part to the fact that lateral tensile stresses have been reduced or changed to compression.

In the quick test hydrostatic pressure is applied to the specimen and it is tested immediately in compression, the entire process requiring less than thirty minutes. If the specimen tends to change volume during the test, and most undisturbed clays do, some of the hydrostatic and deviator stresses are carried

by pressure in the water filling the clay voids which does not have time to flow out. Thus the actual stresses acting on the clay are difficult to evaluate. Sample disturbance increases the tendency of clay to consolidate and increases the error.

In both the consolidated quick and consolidated slow tests the specimens are placed under hydrostatic pressure and allowed to consolidate fully before being tested in compression. If the sample were perfect and undisturbed and the hydrostatic pressure were not greater than the preconsolidation load, little or no consolidation would take place and the test would begin with the soil in its natural condition. The results of consolidation tests show, however, that most undisturbed clay specimens in the laboratory consolidate under their preconsolidation loads. Therefore laboratory specimens under triaxial compression are not tested in their natural condition. As an example, the results of consolidated triaxial compression tests on undisturbed Chicago clay are shown in Table 2, each set of data being the average of two or three tests (21). All

TABLE 2.—AVERAGE RESULTS OF TRIAXIAL COMPRESSION TESTS ON UNDISTURBED CHICAGO CLAY
(All Stresses Are in Kilograms Per Square Centimeter)

Description	Hydrostatic Pressures:									
	0		0.5		1.0		2.0		4.0	
Type of test	Quick	Slow	Quick	Slow	Quick	Slow	Quick	Slow	Quick	Slow
Natural void ratio	0.762	0.762	0.754	0.755	0.743	0.758	0.769	0.765	0.758	0.777
Deviator stress[a]	0.267	0.275	0.540	0.672	0.722	0.768	1.10	0.90	1.62	1.63
Modulus of deformation[b]	26.7	27.5	54.0	67.2	72.2	76.8	110.0	90.0	162.0	163.0
Void ratio[a]	0.762	0.762	0.708	0.685	0.687	0.690	0.635	0.645	0.570	0.575
Poisson's ratio,[a] μ	....	....	0.46	0.31	0.47	0.31	0.48	0.32	0.47	0.28
Stress in pore water[a]	....	....	0.52	0.50	0.69	0.64	1.09	0.78	1.74	1.29
Lateral soil stress,[a] σ_3	....	....	0.02	0.00	0.31	0.36	0.91	1.22	2.26	2.71
Vertical soil stress,[a] σ_1	....	....	0.52	0.67	1.03	1.13	2.01	2.12	3.88	4.34
"Modulus of elasticity"[c]	26.7	27.5	54	67	75	90	114	134	176	282
Maximum deviator stress	0.46	0.42	0.73	0.92	1.26	1.50	1.62	2.64	3.13	5.31
Void ratio at maximum stress	0.762	0.762	0.703	0.670	0.672	0.630	0.587	0.573	0.533	0.512

[a] These are values from tests at 1.0% strain. [b] Slope of the deviator-stress versus axial-strain curve. [c] Computed from Eq. 2.

the specimens in these tests were prepared from one 12-in. cubical undisturbed sample, each specimen being consolidated fully before the test. The "modulus of elasticity," E, was computed from the formula

$$E = \frac{1}{\epsilon} (\sigma_1 - 2 \mu \sigma_3) \qquad (2)$$

in which ϵ = unit strain; σ_1 = vertical soil stress; σ_3 = lateral soil stress; and μ = Poisson's ratio.

The preconsolidation load for this clay was approximately 1.3 kg per sq cm. The data in Table 2 show the decrease in void ratio of good undisturbed speci-

mens under hydrostatic pressures less than the preconsolidation load and the difficulty of separating the effects of change in void ratio and of the applied hydrostatic pressures on the observed increases in strengths and "moduli of elasticity." The increases in test strengths with increasing lateral pressures, which have been attributed to an angle of internal friction in undisturbed clay, may, in fact, be due entirely to decreases in void ratio. Observations by Professor Terzaghi on the field behavior of undisturbed soft clays (22) (23) indicate a zero effective angle of internal friction in natural deposits of considerable depth.

The fact that most undisturbed clay specimens consolidate during triaxial compression tests apparently overbalances the advantage of the test as a method for measuring the strength and elastic properties of soil in its natural condition. It is possible that quick tests and tests under hydrostatic pressures less than the preconsolidation loads may be used successfully on good undisturbed samples to predict actual soil properties. With present methods the possible results do not represent an improvement over the results of unconfined compression tests.

Applications of Results of Laboratory Tests on Undisturbed Samples

The value of the preceding analyses of the effects of sample disturbance on laboratory test results, and indeed the value of laboratory testing of undisturbed samples, depend on the extent to which the results can be applied in practical foundation work. All existing methods for the solution of problems in earthwork and foundation engineering can be placed in one of three categories, each of which uses test results in a different way and to a different extent. In terms of their underlying philosophies these three arbitrary groups can be called empirical, theoretical, and semi-theoretical. If a solution method is based purely on mathematical analysis of an idealized material, it is theoretical; if it is based on theory modified by field observations, it is semi-theoretical; and if it has no theoretical basis or justification, it is empirical. This definition of "empirical" applied to foundation engineering is correct only in regard to the underlying conceptions of the solution methods, some of which (for example, the direct use of unit loads obtained from small-scale field loading tests in foundation design) do not have the support of direct observational evidence.

The true objective of the empirical methods is to establish statistical correlations between soil classes and the field behavior of structures. This is the approach adopted by the Society's Special Committee on the Bearing Value of Soils for Foundations in the early twenties, by highway engineers, and by agricultural soil scientists. Large volumes of data have been accumulated and synthesized into rules for the construction of highways, dams, and foundations and into building code tables of bearing values. The soil classes in these tables and construction rules are identified by verbal descriptions of their appearance, feel, and odor, and by numerical results of simple classification tests performed on remolded specimens. Therefore the application of empirical methods does not require undisturbed samples. The history of every science, and of foundation engineering, shows that progress solely on an empirical basis is painfully slow. F. M. Baron, Assoc. M. Am. Soc. C. E. (24), quoting long-forgotten

engineering articles, demonstrates the fact that empirical foundation design has changed little in the past thirty years and, in fact, has changed only in details in two thousand years. Therefore, the fact that empirical methods do not require undisturbed samples cannot be considered a serious indictment of laboratory testing of such samples.

On the opposite extreme, purely theoretical methods proceed through mathematical analyses of forces, stresses, and strains to numerical solutions for deformations or for limiting conditions of stability. Problems in soils engineering, involving large, continuous, indeterminate masses, require the assumption of simplified physical properties for the soil for mathematical solution. The degree to which the assumed properties deviate from the real soil properties depends on the theory. The "classical" soil theorists, for example, Coulomb and Rankine, assumed that the limiting strengths of soils depended only on friction and cohesion without any reference to strains or deformations. Theoretical methods are increasingly popular today because they are the familiar approach of applied mechanics and because the powerful tools of theories of elasticity and plasticity, which are based on strains and deformations, have become widely known. A simple example is found in theory of elasticity solutions for the settlements δ_c at the centers of loaded areas carried by an infinite depth of homogeneous elastic material, all of which can be reduced to the form:

$$\delta_c = C \frac{P\,b}{A\,E} (1 - \mu) \quad \text{......(3)}$$

in which P is the total load on the area, A and b are the area and width of the loaded area, and C is a numerical constant which depends on the shape and rigidity of the loaded area. Both lateral squeezing and volume change of the soil contribute to settlements. As shown by Professor Casagrande (25), Poisson's ratio, μ, is equal to 0.5 when no volume change occurs and to zero when settlement is due exclusively to volume change. Thus, the equation can be written:

$$\delta_c = C \frac{P\,b}{A} \left(\frac{0.75}{E} + \frac{1.0}{K} \right) \quad \text{......(4)}$$

in which K = the bulk modulus for the soil. In this form, E may be determined from unconfined or triaxial compression tests and K from consolidation tests, both on undisturbed samples. Laboratory tests on undisturbed samples have shown that the "modulus of elasticity" determined from either type of compression test may not be the effective modulus for the natural soil and furthermore that the modulus may increase with increasing depth and confinement of surrounding soil. The theory assumes that both E and K are constants throughout the soil mass. Derivation of an effective bulk modulus for the natural soil from consolidation test results poses a similar problem. Neither the theory nor its applied results are more accurate than are laboratory test results in relation to natural soil properties and both depend on the extent to which assumed physical properties approximate natural conditions. Rigorous theoretical solutions are deceptive because they have the superficial aspect of

precision. Nevertheless, for progress and development in foundation engineering the theoretical method is preferable to an empirical approach which is unrelated to theory because, paradoxically, in a few years, theories have given foundation engineers a means of transmitting to others experience with the actual behavior of soil in construction operations which was lacking after hundreds of years of empiricism. The present situation in soils engineering is precisely parallel to that of structural engineering in the nineteenth century.

Cautious and intelligent use of theoretical conceptions and methods is the hope of the future in foundation engineering. The restrictive assumptions of rigorous theoretical methods mean that they can never be applied safely to soils without full-scale field observations to determine their limitations. This leads to a modified or semi-theoretical method in which rigorous theoretical solutions for idealized materials are used as a guide to the phenomena which may affect field performance and as a basis for field observations. Theoretical solutions are then modified and simplified into a workable form with a degree of accuracy compatible with the accuracy of determination of soil properties from tests. Tests on undisturbed samples are needed in this method both in conjunction with field observations and, later, in applications to design studies, because unpredictable, erratic effects of severe sample disturbance lead to unsafe inconsistencies in the correlation of tests with observations. The test results are corrected as well as possible for effects of minor sample disturbance and other sources of error and are used with the simplified theory. Field observation data sufficiently complete to be used in this way are rare and, in many problems, engineers must still struggle with empirical or theoretical methods supported by hypotheses based on indirect evidence. An outstanding example of the semi-theoretical approach is the Swedish circular-arc method for the analysis of stability of slopes. It is based on adequate field observations and sound mechanics and has been confirmed repeatedly at the expense of purely theoretical methods.

The problem of determining safe bearing values for footing design affords an excellent comparison of these three approaches to foundation analysis. Followers of the empirical or "practical" method consult tables of bearing value for the soil immediately below the footings or perform small-scale field loading tests on that soil and use the results directly in design. The theorists realize that stresses in the soil at considerable depths as well as immediately below the footings affect the performance of the structure. They solve for stresses and deformations by the theory of elasticity. From these solutions bearing values for design are selected which will cause the soil stresses and deformations to remain within acceptable limits, assuming that soil has all the properties of an ideal elastic solid. The semi-theoretical or realistic method accepts the conceptions brought out by the theoretical solution and retains its basic principles but, since soil is neither homogeneous nor elastic, it employs simple approximations to compute soil deformations and limiting footing loads, for example the Terzaghi, modified Prandtl, or Krey methods for approximation of ultimate bearing capacities. Undisturbed sampling and accurate laboratory testing of the samples are a vital part of the theoretical and semi-theoretical methods. The major difference between these two methods in regard to samples and

laboratory testing appears in the way in which the test results are interpreted and used.

Conclusion

Thoughtful evaluation of possible methods for progress leads to the conclusion that laboratory testing of undisturbed samples will play an increasingly important rôle. As rational methods become more reliable the necessity becomes greater for evaluating the effects of sample disturbance on laboratory test results and for extrapolating from the test results to actual soil behavior. The effects of sample disturbance discussed in this paper show that laboratory testing becomes reliable only for samples which have undergone a minimum of disturbance. In the writer's experience only 8-in. to 12-in. cubical samples cut out by hand in test pits fall in this category. If sampling equipment and careful techniques can be evolved which produce samples which are comparably undisturbed, they are well worth additional effort and cost in sampling operations. The most costly sampling operations are those in which large boring samples are obtained in a partly disturbed condition because laboratory tests on such samples yield results that have little relation to reality and in which the effects of previous stress history and actual properties have been so distorted that a reconstruction of the probable true conditions is impossible.

An attempt has been made, in this paper, to extrapolate from observed data to the true natural properties of the soil, with some support from field observational evidence. This method leads to the following conclusions:

(1) Consolidation tests on good undisturbed clay specimens yield results that can be extrapolated to reasonably close approximations of the stress-volumetric properties of natural deposits.

(2) The effects of sample disturbance on direct shear tests have not been evaluated.

(3) Unconfined compression tests on specimens from undisturbed samples cut out by hand in pits or excavations yield compressive strengths that approximate natural strengths reasonably well, but may not produce stress-strain curves or elastic constants comparable with those of the natural soil.

(4) Triaxial compression tests on consolidated specimens of undisturbed clays do not yield properties comparable with those of natural deposits because even a minimum of sample disturbance results in consolidation under lateral pressures to test void ratios smaller than those of the natural deposit.

(5) Laboratory testing of undisturbed samples is valuable in the analysis of engineering soils problems only if its limitations are known and strictly observed.

In the writer's opinion, undisturbed sampling and laboratory testing of undisturbed samples are worthwhile provided:

(6) The economic importance of decisions based on the results of an analysis and the degree of reliability of the solution methods for the particular problem justify the cost of obtaining the best possible samples.

(7) Care is taken that samples undergo an absolute minimum of disturbance.

(8) The differences between test specimens and soil in nature are understood and can be evaluated.

(9) The test results are applied with an understanding of their limitations and the limitations of the solution methods.

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(21) **"Shear Failure of Soils,"** by P. C. Rutledge, *Proceedings*, Fifth Texas Conference on Soil Mechanics and Foundation Eng., February, 1942, Pt. I, p. 18.

(22) **"Stability of Slopes of Natural Clay,"** by Karl Terzaghi, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Cambridge, Mass., Vol. I, June, 1936, pp. 162–163.

(23) **"Earth Pressure and Shearing Resistance of Plastic Clay: A Symposium"**: "Liner-Plate Tunnels on the Chicago (Ill.) Subway," by Karl Terzaghi, *Proceedings*, Am. Soc. C. E., June, 1942, p. 873.

(24) **"The Study of Earths—An American Tradition,"** by Francis M. Baron, *Civil Engineering*, August, 1941, pp. 473–476; and discussions by Karl Terzaghi and A. E. Cummings, October, 1941, p. 614.

(25) Discussion by A. Casagrande of **"Distribution of Stresses Under a Foundation,"** by A. E. Cummings, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 1123–1126.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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REPORTS

ADVANCES IN SEWAGE TREATMENT AND PRESENT STATUS OF THE ART

FIRST PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION ON SEWERAGE AND SEWAGE TREATMENT

With the creation of federal aid through projects of the Public Works Administration (PWA) and Work Projects Administration (WPA), the number of sewage treatment works in the United States has increased markedly in the past few years (Table 1). This increase has occurred principally through the building of many small plants.

The choice of the type of plant has been somewhat widened, with respect to both the treatment of sewage itself and the dewatering and disposal of sludge. The distribution of the different basic types of treatment installed is shown in Table 1. Under items 4, 5, and 6, Table 1(*b*), a number of tanks are included, apparently, which are part of complete plants listed under intermediate and secondary treatment (Table 1(*c*)). The data for 1939–1940 represent actual plants. According to Table 1, the number of sedimentation plants is increasing. In the field of chemical treatment (item 8, Table 1(*c*)) there has been a rapid rise in the number of plants since 1934. The activated sludge plants have increased (item 8, Table 1(*c*)), particularly in the type using mechanical aerators, until in 1940 they were serving a total population of some 10,479,800. Trickling filters (item 10, Table 1(*c*)) have increased also, but in lesser degree. Intermittent sand filters (item 11, Table 1(*c*)) apparently are decreasing in popularity. The application of sewage or effluent to land (item 12, Table 1(*c*)) has increased since 1934. Chlorination (Table 1(*d*)) has also increased, until the sewage or effluents of 14,336,400 people were chlorinated in 1940, or about 35% of the total population served. However, only 20% of the plants have chlorine apparatus.

The expenditure for sewage treatment by various municipalities has been increasing steadily. A brief summary of the cost of the larger installations

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(Table 2) shows approximately the capital sums expended by various municipalities or commissions to December 31, 1940, for sewage treatment and intercepting sewers. The expenditure per capita of census population (as of 1940) ranges approximately from $9.60 to $44.07, according to local conditions and degree of treatment.

TABLE 1.—CENSUS DATA ON SEWAGE TREATMENT IN THE UNITED STATES

No.	Treatment	No. of Plants			Population served; 1939–1940[c]
		1934[a]	1938[b]	1939–1940[c]	
	(a) All Types of Treatments				
1	Total	3,697	4,667	5,580	40,618,000
	(b) Primary Treatment				
2	Less than sedimentation	58	138	49	3,288,300
	Sedimentation Only:				
3	Total	1,937	2,552	2,889	15,097,600
4	Septic tanks	1,420	1,461	1,409	1,462,800
5	Imhoff tanks	1,687	2,062	1,083	4,910,300
6	Separate tanks	475	727	405	8,760,800
	(c) Intermediate and Secondary Treatment				
7	Total	....	....	2,630	22,143,400
8	Chemical	15	114	185	4,012,800
9	Activated sludge	104	223	302	10,479,800
10	Trickling filters	789	1,140	1,486	8,424,700
11	Intermittent sand filters	606	524	432	904,700
12	Application to land	213	193	304	852,000
13	Miscellaneous	127	109	151	2,217,700
	(d) Chlorination				
14	Total	655	921	1,127	14,336,400

[a] *Engineering News-Record*, August 15, 1935. [b] *Engineering News-Record*, January 19, 1939. [c] Data released through the courtesy of the U. S. Public Health Service, January 14, 1942.

SEWAGE RESEARCH

The annual review of the literature on sewage and waste treatment and stream pollution by the Committee on Research, Federation of Sewage Works Associations (1),[1] summarizes the art clearly each year from the research, laboratory, and operating standpoints. The Society Committee, therefore, has not undertaken a comprehensive review but has sought to indicate progress and trends in various fields of sewage treatment from a broad standpoint, with occasional emphasis on certain details of interest to the engineer, either in design or operation.

Considerable research work is still being undertaken in three main fields: Laboratory, pilot plants, and full-scale tests. In the laboratory, research is

[1] Numerals in parentheses, thus: (1), refer to corresponding references in the Bibliography (see Appendix).

under way at many plants, as well as in completely equipped groups, such as the New Jersey Agricultural Experimental Station in the field of sewage treatment, and the U. S. Public Health Service in the field of stream-pollution

TABLE 2.—CAPITAL EXPENDITURES ON SEWAGE TREATMENT BY MUNICIPALITIES IN THE UNITED STATES, TO DECEMBER 31, 1940 (DOLLARS)

No.	Municipality	Sewage treatment	Intercepting sewers	Total
1	The Sanitary District of Chicago (Ill.)	79,600,000	87,400,000	167,000,000
2	New York, N. Y.	33,500,000	13,000,000	46,500,000
3	Boston (Mass.) Metropolitan District	....	36,984,000[a]	36,984,000
4	Detroit, Mich.	10,459,400	15,405,500	25,864,900
5	Cleveland, Ohio	17,319,000	6,535,000	23,854,000
6	Milwaukee, Wis.	11,838,136	8,425,400	20,263,500
7	Minneapolis-St. Paul (Minn.) Sanitary District	4,293,000	11,462,000	15,755,000
8	Buffalo, N. Y.	....	....	15,000,000
9	Los Angeles, Calif.	1,614,541	12,761,500	14,376,000
10	Baltimore, Md.	10,370,000	3,900,000	14,270,000
11	Los Angeles County (Calif.) Sanitation District	1,142,000	10,719,000	11,861,000
12	Columbus, Ohio	5,676,350	5,437,000	11,113,350
13	Toledo, Ohio	2,125,700	6,323,600	8,449,300
14	Akron, Ohio	4,300,000	3,750,000	8,050,000
15	Fort Worth, Tex.	1,884,000	3,488,800	5,372,800
16	Hartford, Conn.	1,490,012	3,796,233	5,286,245
17	Fort Wayne, Ind.	....	....	5,200,000
18	Indianapolis, Ind.	2,469,000	1,540,000	4,009,000
19	Atlanta, Ga.	1,980,000	1,980,000	3,880,000
20	Denver, Colo.	3,000,000[b]	700,000[c]	3,700,000
21	Duluth, Minn.	1,500,000	1,750,000	3,250,000
22	Springfield, Mass.	1,490,000	1,365,000	2,855,000
23	Peoria, Ill.	....	....	2,822,000
24	Springfield, Ill.	....	....	2,268,400

[a] Includes the Boston Main Drainage System and the North and South Metropolitan systems. [b] Includes $1,500,000 for the sewage treatment works and $1,500,000 for a tunnel to bring in diluting water. [c] This is the city's share in a WPA project.

control. Since The Sanitary District of Chicago (hereinafter, for convenience, referred to as Chicago) discontinued various scale tests at its West Side plant in 1939, considerable work with pilot plants has been conducted on industrial wastes at Lansing, Mich., and Madison, Wis. Continued activity is shown in full-scale tests in various sewage treatment works of all sizes to determine the best way in which to operate efficiently and produce a suitable effluent at as low a cost as possible.

PUBLIC HEALTH SERVICE

In 1939 the U. S. Public Health Service was placed under the Federal Security Agency. Its powers have been broadened, especially in emergency health and sanitation work for the civil population in the present national situation.

The Ohio River Pollution Survey was completed in 1941, a cooperative investigation between the U. S. Public Health Service and the U. S. Army Corps of Engineers, under the general direction of the Ohio River Committee. An interim report on the middle third of the Ohio River watershed was submitted by the U. S. Public Health Service to the Committee in 1940. Other

surveys were made on the North Canadian, Grand (Neosho), and Scioto rivers. A census of all the sewage works in the United States has been completed.

Research has continued on the biological and biochemical factors underlying the oxidation and assimilation of organic matter in liquids. The predominant bacteria have been studied in the activated sludge process and trickling filters.

National Legislation

The Stream Pollution Bill passed both Houses of Congress in 1940, but failed to become a law because of disagreement between the two Houses as to whether or not Federal abatement of new pollution should be authorized. Two Interstate Antipollution Covenants were authorized by Congress in 1940, one on the Ohio River and the other on the Potomac Valley.

Sedimentation

In the field of sedimentation, one development is the increased number of types of mechanisms offered for removing sludge from settling basins; another is the size of the units. The plants of the Minneapolis-St. Paul (Minn.) Sanitary District, and of Detroit, Mich., are examples of the largest rectangular basins for sedimentation of raw sewage (2)(3) with straight-line, sludge-removal mechanisms. The tank units at the former location have a length of 290 ft, a depth of 15.5 ft, and a width of 56 ft, divided into three passes, each 18 ft wide; and at the latter location they have a length of 270 ft, a depth of 14 ft, and a width of 117 ft, divided into seven channels, each 16 ft wide. The detention periods are 84 min and 90 min, respectively, for plain sedimentation.

In the rotating mechanism types, the largest units in preliminary treatment of sewage are 170 ft in diameter, at Baltimore, Md. (three units each handling 25 mgd).

For the final settling tanks in activated sludge treatment works, circular tanks have been built up to 126 ft (inside diameter) at Chicago, Ill., and Baltimore. Most of the rotating mechanisms utilize squeegees. The Tow-Bro type that utilizes hollow rotating arms equipped with multiple inlets is installed in twenty-four cities, of which Milwaukee, Wis., is the outstanding example, with four final tank units on activated sludge, each 98 ft in diameter. Rectangular tanks with straight-line cleaning mechanisms are provided in the larger plants, such as the Wards Island, Tallmans Island, and Bowery Bay (in New York, N. Y.), and Columbus, Ohio, as well as in smaller plants. The straight-line, sludge-removal mechanism in the Bowery Bay (40 mgd) activated sludge plant represents a departure from current practice, in that the bottom scraping flights travel in the direction of the sewage flow and at a velocity two or three times that usually used.

The tests conducted by G. E. Hubbell, Assoc. M. Am. Soc. C. E., for Detroit (4) on both rectangular and circular tanks have added to the knowledge of the sedimentation of raw solids. However, there is still need of further research on the settling of activated sludge, particularly in developing the larger designs, as shown by the experience at the Southwest Works in Chicago. A center-feed radial flow tank, 75 ft in diameter, proved to be very satisfactory at the North Side Works. Development of the tank units to 126 ft in diameter

at the Southwest Works produced a tank with peculiar eddy currents at the inlet and poor settling qualities. The tank efficiency has been improved by adding an inner circular trough 4 ft wide, 4 ft inside the outer wall, and eliminating the original peripheral weir. At Baltimore, however, no operating difficulties were experienced with the 126-ft diameter tank. The vagaries of the center-feed tank apparently are not yet under complete control for larger units. It would seem as though the development of mechanical devices for removing sludge has outstripped the control of the settling of activated sludge.

Self-Flocculation

To improve the settling qualities of sewage, self-flocculation is being studied, both in the laboratory (5) and in the plant (6). Periods suggested by H. W. Gehm (5) are a maximum of 40 min for flocculation, followed by a minimum of 1 hr for settling.

Activated Sludge

Activated sludge plants are being constructed where a high-grade effluent is required. At present there are more than 106 plants in the United States blowing air through diffusers. When it is completed the West-Southwest Works (Chicago) probably will rank as the world's largest plant for a long time, with a nominal average capacity of 800 mgd. The part already constructed is still in the tuning-up stage. Among the details requiring attention were the final settling tanks, the adjustment of the steaming capacity of the boilers to the drying capacity, and the enlargement of the coal crushers to furnish coal to develop the heat for drying the sludge for fertilizer.

The various trends in the operating procedure of activated sludge plants have been discussed in detail in the Report (7) of the Committee on Sewage Disposal of the American Public Health Association. Among the more outstanding trends at the larger plants are the use of a smaller quantity of air, with a shorter period of aeration, producing a minimum of nitrates instead of 5, 10, or even 15 ppm as was the case some years ago; carrying a lower suspended solids content in the mixed liquor; and maintaining a minimum depth of sludge in the final settling basins. The use of the sludge index has proved helpful in maintaining operating control.

Distributed Loading

The studies of G. M. Fair, M. Am. Soc. C. E., and J. E. McKee, Jun. Am. Soc. C. E. (8), at Harvard University, in Cambridge, Mass., and R. H. Gould in New York City on distributed loading of activated sludge plants are of interest. As yet, few data are available, except on a laboratory scale.

Diffusers

The behavior of diffuser plates at activated sludge plants serving different cities has been studied carefully (7). Apparently, where the sewage is low in ferrous iron, little difficulty with clogging is reported. Where sewage contains ferrous iron in considerable amount (perhaps more than 5 ppm), clogging has occurred within a year, and, where the iron content is greater than 50 ppm,

clogging may be more rapid. At a number of the iron-free plants, diffusers have been in service for 10 to 15 years with but little attention—sometimes no attention. At plants where there is considerable iron in the sewage, acid cleaning may be required as often as every 2 to 4 weeks. The source of the iron is generally spent acid pickling liquor from steel mills or foundries. One of the unexpected difficulties at Chicago has been the behavior of the diffusers at the Southwest Works. Originally, these had a permeability rating of approximately 40 cu ft of air per minute. Clogging began to develop after 1 year, necessitating cleaning. The iron content of the Southwest sewage is low, averaging approximately 5 ppm. At the Calumet plant in Chicago, the iron content originally was high. The yearly average for 1936 was 17 ppm, and for 1937, 13 ppm, with a maximum hourly rate of 270 ppm. Clogging ensued. A reduction in the iron content lengthened markedly the period between cleanings. Now clogging is reappearing. Serious clogging has occurred at Cleveland, Ohio (within 1½ to 2 years of installation), Wards Island, Milwaukee (within 5 years), Indianapolis, Ind., Lima, Ohio, Houston, Tex., and elsewhere. Most of the clogging reported has been accompanied by a high iron content in the sewage, although Wards Island reports only 1.5 ppm; yet at Milwaukee, one half the original plates, with a permeability rating of approximately 12 to 15 cu ft per min, are still operating satisfactorily with one cleaning by spalling during 15 years. At the North Side plant in Chicago, more than 90% of the original plates, with a rating of approximately 12 to 15 cu ft per min, are still operating satisfactorily after 12 years without cleaning. The exact cause of the trouble is not clear. The various plants interested are cooperating in an effort to find out. The methods that have been tried to relieve clogging are many (9), but are not always permanent in effect.

Mechanical Aerators

For the smaller municipal and institutional plants, mechanical aerators are increasing in popularity. There are probably in excess of 131 installations in the United States. A number of types are available, with a central rotating device. The British types, one with paddle wheels in long, shallow channels (Haworth), and one with propellers (Hartley), are practically nonexistent in the United States.

Treatment of Excess Activated Sludge

In 1939, among the 220 activated sludge plants in the United States, only sixteen lacked sludge-digestion tanks. Today five plants are dewatering and heat-drying sludge for sale as fertilizer (Chicago—two plants; Houston; Milwaukee; Pasadena, Calif.). However, Pasadena is reported to be preparing to abandon heat-drying and to substitute ocean disposal.

Trickling Filter

The low-rate trickling filter is still very much the "old reliable" and is holding its own where a high-grade effluent is required. For smaller plants, in combination with settling tanks, it is still in favor. Better means of dis-

tributing sewage are sought. Circular filters with revolving distributors are growing in favor.

Of the new trickling filter installations, that at Dallas, Tex., appears to be the largest (10), with sixteen units, each 176 ft in diameter, designed to handle 48 mgd. This is designed so that two-stage treatment is possible, as well as recirculation. Covered filters have been built at Hibbing, Minn. (11); Pennsylvania State College, State College, Pa.; and Johannesburg, Union of South Africa (12).

The most recent trend of development has been in high-rate filtration, recirculation of effluent, and stage filtration. The growth of trickling filter installations in the United States has been as follows:

Year	No.
1926	371
1937	947 (73)
1938	1,140 (29*a*)
1940	1,486 (74)

High-Rate Filter

Beginning in 1936, among the smaller plants, high-rate filters have been installed in increasing numbers, and have been operated with a loading as great as ten times the normal. Although relatively good effluents are generally claimed, it appears probable that, with one-stage filters, the device is more suitable where a lower grade effluent is sought than the effluent of a normal trickling filter or activated sludge plant. Two-stage or three-stage treatment is said to develop a better grade of effluent.

An inherent principle is to secure as uniform a rate of application of the liquid to the surface of the filter as possible. Single-stage and two-stage filters are in use, the latter being preferred when treating very strong sewages. If desired, a part or all of the effluent may be returned for circulation.

Some designers advocate a larger size of medium ($2\frac{1}{2}$ to 4 in.) in the primary stage of two-stage filters or in single filters, with finer stone (2 to 3 in.) in the secondary stages of a two-stage filter. Others advocate $1\frac{1}{2}$-in. to $2\frac{1}{2}$-in. stone in the primary filter and $\frac{3}{4}$-in. to 1-in. stone in the secondary filter. Distributors are available with rotating disks or rotating arms. Rates of dosing between 100 and 125 million gallons per acre per day (mgad) are stated (13) to have been used when recirculation is practiced, whereas on single-stage filters rates up to 30 mgad are used, without recirculation.

Where cold winters occur, the high-rate filter is easier to cover than the standard low-rate filter, because of its smaller area.

High-rate filters appear to have a considerable field of application for treating the stronger industrial wastes (14), as well as those for the smaller municipalities or institutions. The largest installation of the Jenks type is at Fort Read, B.W.I. (5 mgd), and the largest Halvorson-Smith type is at Lubbock, Tex. (4 mgd). Altogether, at the end of 1941, approximately 137 plants of both types have been installed.

Chemical Precipitation

The furor over chemical precipitation current between 1930 and 1933 has now subsided, as the various procedures experienced the test of unbiased appraisal. Today, chemical precipitation is rated as a process producing an effluent of higher grade than plain settling, but in general not equal to that of an activated sludge or trickling filter plant. The seasonal use of chemicals is established to increase for a few months of the year the performance of a settling plant, as at the Minneapolis-St. Paul Sanitary District, Waukegan, Ill., Colorado Springs, Colo., and Coney Island, N. Y., and various plants on the Raritan River in New Jersey.

The choice of chemicals has largely narrowed down to ferric sulfate, ferric chloride, and chlorinated copperas. The value of sodium silicate is being studied in combination with chlorinated copperas or alum. The use of sulfate of alumina is increasing in popularity. Comparative tests (15) at Waukegan indicate that, per million gallons, copperas combined with sodium silicate is the cheapest ($2.27). Next in order are sulfate of alumina ($3.22), ferric chloride ($3.24), and chlorinated copperas ($3.27).

Guggenheim Process

The revised Guggenheim process has been in operation at New Britain, Conn., since 1937. At present there are in operation four municipal plants and three on industrial wastes, making a total of seven. Under construction are two municipal plants. The process appears to be favored where metallic salts of industrial origin are prevalent in the sewage. A large-scale test of the Guggenheim process is being undertaken at the Wards Island plant in New York City to furnish data directly comparable with the activated sludge process.

Putnam Process

Thus far, the Putnam process has been installed at four places, mostly small towns, since 1933. In the largest town, St. Charles, Ill. (population 5,870), the process has been abandoned as such. This process includes chemical precipitation, the resulting sludge being retorted and burned to a char for use as a precipitant. The claim that the distillates can be sold as an insecticide is unsubstantiated.

Chlorination

The use of chlorine for sewage treatment was discussed in the Report of the Committee on Sewage Disposal, Public Health Engineering Section, American Public Health Association (16), in 1933. The conclusions expressed therein are still fundamentally sound. In some details the practice has broadened in such fields as the control of the bulking of activated sludge, sludge concentration, and chemical coagulation. A new development is the increased removal of grease by using chlorine in pre-aeration tanks (17)(18)(19). There is still a need for research on the fundamentals underlying the action of chlorine.

Two large chlorination plants were put in service recently for treating effluent at Detroit (3) (capacity, 27 tons per 24 hr) and at Buffalo, N. Y.

(capacity, 15 tons per 24 hr). In the latter, the chlorine demands are very variable (20). At Detroit, Buffalo, and Minneapolis-St. Paul, manually controlled or semi-automatic vacuum-type solution-feed chlorinators are provided, each having a capacity of 6,000 lb per day. At Cleveland, 4,000-lb-per-day chlorinators are installed.

At Detroit (3) the chlorine is bought in tank cars of either 16-ton or 30-ton capacity, and is removed from the cars as a liquid. Dry compressed air is introduced into the cars to force the chlorine out through 1-in. lines of extra heavy black steel pipe. A telltale device is provided, consisting of an ordinary 150-lb chlorine cylinder on a scale equipped with an alarm. City water is used in preparing the chlorine solution.

In the more recent designs for the larger plants, greater attention is given to safety features than heretofore. At Detroit, to guard against excessive pressure in the evaporators, these and the telltale are equipped with frangible silver disks, which rupture at 225 lb per sq in. and discharge into pipes leading to the outside.

New Process

Among the more recent offerings is the Hayes process, a combination of settling and contact aeration in one or more stages. This device apparently belongs to the group of so-called contact aerators. It is reminiscent of the process tried by H. W. Clark in 1913 (21)(22) at Lawrence, Mass. The Hayes plants so far have been installed on relatively low flows at small towns and a few cantonments. Published data on design and performance are lacking. For the contact surfaces, broken stone, thin aluminum plates, transite sheets, and concrete cylinders have been tried. The surface medium is set above a hopper bottom tank, from which the resulting sludge is removed.

The Committee is aware of U. S. Patent No. 2,154,132, issued to E. B. Mallory, for a process of controlling the purification of sewage, and also certain other patents issued to him for sewage purification apparatus—namely, U. S. Patents Nos. 2,204,093, 2,223,257, and 2,223,258. From a published description (23), his method of design and operation apparently is based on mathematical formulas, relating, in part, to the concentration and the percentage of return sludge and the proportions of the aerating and settling tanks.

Grit Chambers

With the introduction of mechanical dewatering and incineration of sludge, some engineers believed that a preliminary settling tank in an activated sludge plant might also serve as a grit chamber, with the combined disposal of grit and preliminary sludge. This was tried at the Calumet (Chicago) Works without success. During a heavy storm the grit entered the settling tank so fast that it stopped the clarifier mechanism. Further, the grit increased the maintenance of the conveyers handling sludge cake. Hence, a separate grit chamber was constructed ahead of the preliminary tanks.

Grit removal and the design and practices in grit chambers, as well as grit washing, are discussed in three papers read before the sanitation group of the

American Society of Mechanical Engineers (24)(25)(26). A method for control of velocities by a flume or other control section has been suggested (27). The tests made by Mr. Hubbell (28) for Detroit on grit-chamber models indicate a useful approach to the problem of design.

Sludge Disposal

Most of the developments in sludge disposal during the past four years have been in vacuum filtration, heat-drying, and incineration.

According to the 1939 inventory of the *Engineering News-Record* (29), 3,170 of the 4,667 plants then treating sewage used air-drying beds. Of these, 223 were covered. The mechanical dewatering installations increased from nineteen in 1936 to sixty one in 1939. The number of incinerator installations increased from six in 1935 to twenty nine in 1939, and approximately thirty seven in 1941.

Available figures in 1941 indicate ten installations using the flash-drying system and twenty seven using the multiple-hearth system. However, the flash-drying system is apparently handling more than twice as much filter cake as the multiple-hearth system and about 20% more dry solids. Where a heat-dried sludge is desired for fertilizer, the flash-drying system apparently leads the field with an output of more than 58,000 tons per yr, as against 25 tons for the multiple-hearth system. Of the multiple-hearth type, three plants are handling garbage and sewage sludge, and an additional three are equipped to do so.

The disposal of sludge is an important part of the sewage problem. As a sewage plant practically never makes a profit, the engineer must attempt to reduce the deficit. If the choice of the method of sludge disposal is based on the most economical plan, the deficit may be kept at a minimum.

Ocean Disposal

Along the seacoast, opportunity is offered for ocean disposal that is denied the inland territory. In the United States, about 1909, Providence, R. I., was the earliest to utilize ocean disposal, at first for press cake from chemical precipitation, then for sludge from plain sedimentation, and more recently for sludge cake from its activated sludge plant. From 1924 to date (November, 1942), the Passaic Valley Sewage Commission (30) utilized ocean disposal for fresh settled sludge. Beginning in 1937, New York City (31) has disposed of preliminary and excess activated sludge in the ocean. Later, the Joint Meeting of Municipalities in Essex and Union counties, New Jersey, disposed of concentrated fresh sludge in the same manner (32). At New Haven, Conn. (33), plain sedimentation sludge was disposed at sea for approximately $9\frac{1}{2}$ years. Owing to interruption of service by ice-locking in the harbor or freezing of the sludge in the decantation tanks, a sludge dewatering and incineration plant was installed, which went into service in July, 1940.

Lagooning of Sludge

Lagooning of sludge, both primary and activated, has been the practice at Indianapolis in a plant serving a population of 386,170. Originally constructed

as a temporary measure, lagoons have been used for more than 15 years, increasing in area to a total of 52.5 acres. The present volume is approximately 33 cu ft per capita. Good results have been obtained by seeding the incoming sludge adequately with supernatant liquor as well as digested sludge. A sandy soil has been helpful in promoting seepage. At Chicago, lagooning is in use at the Southwest Works as a temporary measure until permanent equipment can be obtained for dewatering and heat-drying.

Imhoff Tanks

Imhoff tanks are still popular, particularly for small installations where a minimum of operation attention can be given. The West Side Works at Chicago is the largest installation in the world, with 10,532,000 cu ft of available sludge-digestion space. This plant serves 1,487,773 people, with a flow of about 360 mgd. The excess space available over that required for its tributary population is now used for digesting the preliminary solids from the adjacent Southwest Works.

Sludge Digestion

Among the plants using sludge digestion, there is still need for the correlation of operating records to evaluate various factors and their influence on design. Stage digestion is still practiced at the Los Angeles (Calif.) Sanitation District. A three-stage digester has been installed at Topeka, Kans., for activated sludge (34). A two-stage digester is used at Aurora, Ill. (35), as well as at Iowa City, Iowa (36), Rahway Valley (New Jersey) Joint Meeting Sewage Treatment Works (37), and elsewhere.

Vacuum Filtration

For the most part, the vacuum filtration of sewage sludge is accomplished on cylindrical filters with compressed-air cake discharge. A few so-called "string" filters are used also. The rotating-disk type has practically disappeared from the sewage field. Until 1940 most of the cylindrical vacuum filters on sewage sludge have used a standard copper drainage screen. Following recent developments in the metallurgical field, diagonal drainage grids of wood are being tried at the Calumet and Southwest Works (Chicago). The question of economics usually will determine how small an installation is practicably desirable.

Sludge Concentration

The concentration or thickening of sludge is becoming more prevalent, not only where vacuum filtration is to follow, as at the Southwest plant (Chicago) and at Columbus, but also where ocean disposal of sludge is practiced, as at New York City and Elizabeth, N. J. In many cases, storage is considered to be sufficient. Continuous operating tanks have been tried at the Southwest Works, Chicago, with some degree of success. Various devices for increasing concentration are offered. However, there is still need for comparative research and continued operating data.

Sludge Conditioning

The conditioning of sludge for vacuum filtration depends on the type of sludge. With fresh or digested sludges, the use of elutriation seems to be increasingly popular as a step prior to chemical coagulation. Ferric chloride or ferric chloride and lime are the favored conditioning agents, although aluminum salts are finding a place. Ferric chloride, or occasionally chlorinated copperas, is most commonly used with undigested activated sludge.

Sludge Elutriation

Among the more recently developed procedures, sludge elutriation, prior to filtration, has been attaining importance until at present there are twelve plants so equipped in the United States, serving approximately 3,000,000 people. Among the larger installations are those at Washington, D. C., Hartford, Conn., Springfield, Mass., Baltimore, and Detroit. Elutriation is being provided for digested primary and chemically precipitated sludge at the enlarged Coney Island plant of New York City. Basically, elutriation is a process of washing digested sludge to remove some of the soluble materials which inhibit coagulation, thereby improving the filtering qualities and lessening the cost of conditioning. Where lime and ferric chloride would be the indicated conditioners for digested sludge, elutriation may eliminate the use of lime and thus reduce the cost of vacuum filtration. The deposition of lime carbonate in the filter cloth, in the screen backing for the cloth, and in all the filtrate piping is causing much trouble that can thus be prevented. At Weida in Saxony, Germany, Karl Imhoff, M. Am. Soc. C. E., is reported to have treated a difficult raw sludge containing tannery waste on vacuum filters with the aid of elutriation.

Activated Carbon

The use of activated carbon in the field of sewage treatment appears limited (38) and is subject to conflicting comments. Several observers (39) report that its use increases the drainability of digested sludge, whereas at Aurora, Batavia, and Geneva, Ill., it is reported (42) as ineffective. Research (40) has indicated an increase in gasification of mixed fresh and activated sludge. On the other hand, 8% of activated carbon (on the basis of volatile solids) is reported (41) to have retarded digestion.

Alum as an Aid to Sludge Drying

Alum has been revived as an aid in draining digested sludge, but at Lancaster, Pa. (39), the time of drying was not shortened. At Aurora (42)(43) it proved effective, particularly on sludges containing less than 9% moisture. With a solid content of 6.9%, 1 lb of alum is used for 93 gal of sludge.

Use of Sludge as Fertilizer

A comprehensive survey of the use of sludge as fertilizer (44) was prepared in 1937 by the Committee on Sewage Disposal of the Public Health Engineering Section of the American Public Health Association, which covered the funda-

mentals of the use of fertilizers and the practice in the application of such material to agriculture. The conclusions expressed therein are sound today.

Since 1939, the Calumet and Southwest Works in Chicago have been marketing heat-dried activated sludge in bulk to mixers of commercial fertilizers. This material is prepared in flash dryers. Originally in a finely divided (powdery) state, the physical appearance has been modified to a finely pelleted (granular) material to meet the requests of buyers and overcome certain problems in shipping and storage. Milwaukee, also, has modified its product to a more finely divided material. Pasadena is reported to be abandoning the heat-drying of activated sludge for disposal in the Pacific Ocean via the interceptors of the Los Angeles County Sanitary Districts. The output of heat-dried, activated sludge has increased to approximately 112,000 tons in the United States (see Table 3).

TABLE 3.—QUANTITY OF HEAT-DRIED ACTIVATED SLUDGE SOLD[a]

(2,000-Lb Tons)

Year	Houston	Milwaukee	Pasadena	Chicago	Total
1927	....	20,258	....	....	20,258
1928	....	30,866	839	....	31,705
1929	....	31,602	2,533	....	34,135
1930	....	29,311	3,352	....	32,663
1931	....	26,344	3,385	....	30,243
1932	2,010	37,395	3,892	....	43,297
1933	1,992	34,818	3,266	....	40,076
1934	2,083	32,088	2,630	....	36,801
1935	2,004	31,523	3,546	....	37,073
1936	1,243	46,726	3,304	....	51,273
1937	2,062	49,223	3,518	....	54,803
1938	1,469	42,720	3,625	....	47,814
1939	1,209	45,748	4,149	11,303	62,409
1940	1,528	48,026	4,244	58,464	112,262
1941	1,900[b]	49,215[c]	4,300[b]	56,597	112,012

[a] In 1931, Indianapolis produced experimentally, for one year only, 514 tons. [b] Estimated. [c] Actual ten months plus estimated two months.

Because of the disturbances of world trade by World War II, the demand for heat-dried activated sludge has continued, despite the general decrease in the price of organic fertilizer material for the past 20 years. The extent of the use of materials of sewage origin for fertilizer is governed somewhat by the tonnage of organics used as feed for animals. If conditions change so that fewer animals are fed, higher grade material used for feed finds its way into the lower grade market. Consequently, it is conceivable that, if the market were glutted with offerings, it might be more economical to burn heat-dried activated sludge than to sell it for fertilizer. Therefore, any city contemplating entering the field of commercial marketing of heat-dried activated sludge should provide a plant not only for making fertilizer material, but also for burning or otherwise disposing of such sludge if necessary.

Air-dried digested sludge is apparently increasing in use and is serving as a substitute for manure (44)(45)(46). As such, it can be transported economically only a relatively short distance—perhaps 25 miles.

Heat-drying of air-dried or other dewatered digested sludge does not progress very rapidly, chiefly because of the low grade of the material and the low percentage of available nitrogen (approximately 40%) as compared with the higher percentage in activated sludge (from 67% to 84%) (47)(48).

The fertilizing properties of fresh sludge and digested sludge and the value to crops of the various mineral and organic constituents have been discussed by workers in many localities. Only careful comparative tests will substantiate many of the claims.

Use of Sludge Gas

The earliest utilization of sludge gas in the United States appears to have occurred at the Peachtree Creek plant at Atlanta, Ga., in 1913. The gas was collected and used for lighting, heating, and laboratory purposes. Since the installation of two small (15-hp) gas-driven air compressors at Plainfield, N. J., in 1926 (49), many activated sludge plants have followed this practice until, at Tallmans Island, New York City, two 850-hp gas-driven blower units are the largest now in use. In other plants the gas is used for heating, generating power, and pumping. The largest installation is at Washington, D. C., where a 1,200-hp gas engine drives a generator.

Sludge Friction

The friction losses of sewage sludge flowing through pipe lines were studied in 1929 by a Committee of the Sanitary Engineering Division of the Society (50). The Committee concluded, on the basis of sludge flowing in 6-in. to 12-in. pipes, that the sludge friction losses increase with a decreased moisture content and tend to increase with lower temperatures. More recently, considerable data have been obtained that modify some of the results and clarify the problem.

The friction losses which were originally determined at Chicago in Imhoff tanks were based on sludge that was repumped several times. These losses were found to be low when compared with subsequent tests on sludge pumped by ejectors through 400 ft of 8-in. cast-iron pipe. With sludge containing 87.6% to 89.5% moisture at a temperature of 49° F, *C* in the Hazen-Williams formula varied from 25 to 69 and averaged 47. During one test, *C* was 25 when the moisture in the sludge was 85.3%, and in another test *C* was 9 when the moisture was 83.2%; *C* varied from 102 to 136 and averaged 117 when sewage flowed through the pipe.

Further tests in 1931 at the West Side plant (Chicago) were made with a mixture of activated sludge from the North Side plant and about one third its volume of fresh sludge, at a temperature of 65° to 70° F, when flowing through 1,100 ft of new, straight, 6-in., cast-iron, bell-and-spigot pipe. The loss of head varied with the moisture content (Fig. 1). Other tests were made (Fig. 2) with Imhoff tank sludge flowing through 400 ft of 8-in. cast-iron pipe that had been in service for 5 years. The moisture content of the sludge varied from 80.8% to 89.9%. In general, the Chicago data indicate that sewage sludge containing not more than 3% solids will flow with approximately the same friction loss as water.

In 1933 W. Merkel (51) discussed the flow properties of digested sludge in 20-cm pipes for various concentrations, analyzing tests at Chicago and at Nürnberg and Stuttgart, Germany. In 1938 W. D. Hatfield (52) discussed the viscosity or pseudo-plastic properties of sewage sludges, pointing out the relation of solids concentration and viscosity, and the effect of temperature on the apparent viscosity. In 1939 H. E. Babbitt, M. Am. Soc. C. E., and D. H. Caldwell (53) analyzed the laminar flow of sludges in pipes with especial reference to sewage sludge. Their experiments were based on relatively short pieces of 1-in., 2-in., and 3-in. pipe, although their text utilized data

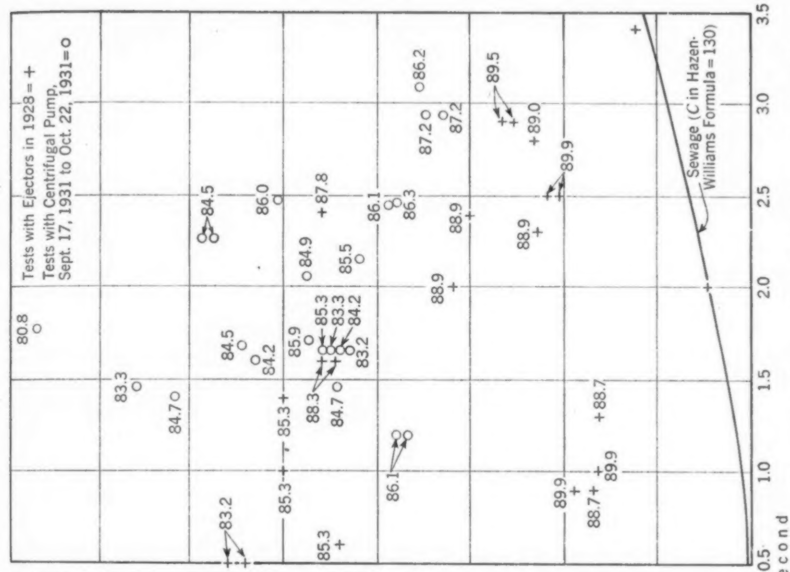

FIG. 2.—FRICTION OF IMHOFF TANK SLUDGE IN 8-IN. CAST-IRON BELL-AND-SPIGOT PIPE; CALUMET SEWAGE TREATMENT WORKS

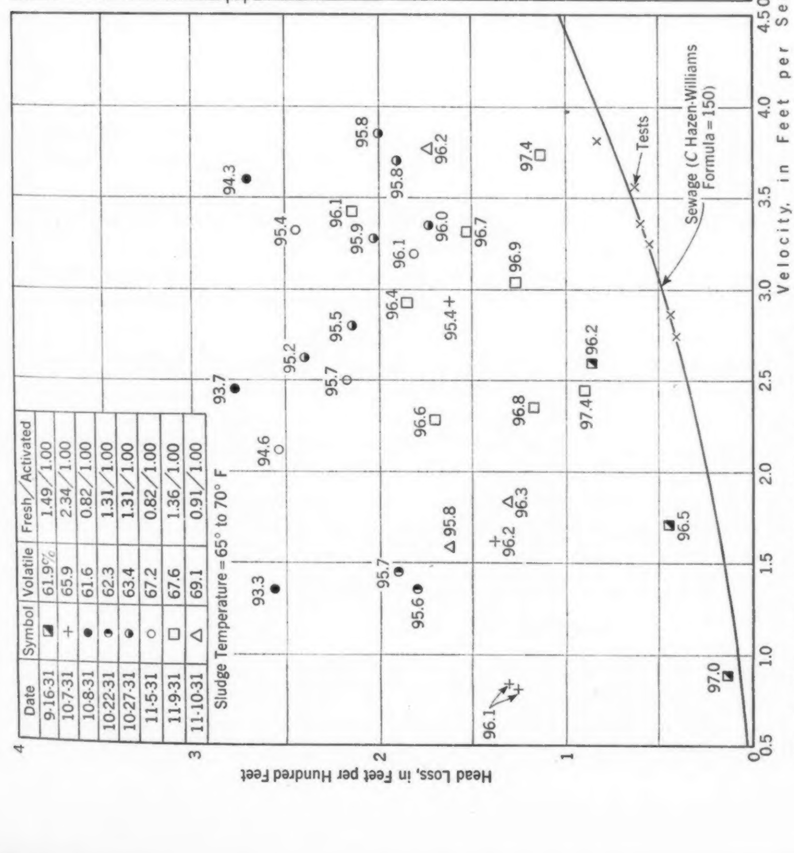

FIG. 1.—VELOCITIES AND HEAD LOSSES FOR COMBINED FRESH AND ACTIVATED SLUDGE IN NEW 6-IN. PIPE; WEST SIDE TREATMENT WORKS

from experiments on pipes $\frac{3}{8}$ in. to 12 in. in diameter. Their analysis shows a theoretical formula for laminar flow and formulas for the upper and lower critical velocities. In conclusion, they recommend further research on the factors affecting the turbulent-flow, frictional losses of sludge flowing in circular pipes, "* * * as the indications are that the common hydraulic formulas for flow of water are not applicable to the flow of sludges."

In 1940 Messrs. Babbitt and Caldwell (54) discussed the turbulent flow of sludges in pipes, from which they conclude "* * * that most sludges, such as drilling muds, clay, slurries, sewage, sludges and other aqueous suspensions of fine particles, when flowing in a pipe at a velocity greater than the critical, follow the fundamental laws of the flow of true liquids." In 1940 W. Rudolfs, M. Am. Soc. C. E., and L. E. West published (55) tests on the pumping of sludge at Elizabeth through a 24-in. pipe 4,400 ft long. The sludge concentrations varied from 5.2% to 8.4% of solid, the temperature of the sludge ranged from 48° to 73.5° F, the ash content ranged from 21.4% to 40.1% of the dry solids, and the average total operating head on the pumps was from 14 to 42 ft. The results show a material increase in loss of head with increased solids concentration, with the rate of loading and calculated viscosities following the general trend. The temperature of the sludge exerted a greater influence than the concentration of solids, the loss of head increasing with decreasing temperature. With the higher ash contents, the rate of flow increased.

Sludge Friction; Activated Sludge

The longest force main in the world through which activated sludge is pumped is at Chicago. This is a 14-in., centrifugal cast-iron pipe with bell-and-spigot joints, about 17 miles long. The exact internal diameter is 14.55 in. A mixture of primary and activated sludge is pumped, averaging about 1.18% solids by weight. When pumping 2.033 mgd of sludge during the original tests in 1928, *C* in the Hazen-Williams formula was 141. The leakage was 0.81 gal per min per mile. In 1941 *C* had decreased to 93 when pumping 1.60 mgd. Examination of a section of the pipe showed a coating of material, largely iron hydroxide and free sulfur, around the upper two thirds of the interior of the pipe. The bottom third was clean. The leakage from the pipe line had increased to 3.13 gal per min per mile, or about four times the original amount, in 1928. The air-relief valves originally installed have not been used.

At Cleveland, the friction loss in a 12-in. sludge force main, which is about 13.2 miles long between the Easterly and Southerly plants, increased markedly between July 19, 1938, and January, 1941. Approximately, *C* dropped from a value with clean water flowing to a value of 76. The pipe was cleaned with a go-devil at the end of January, 1941, and the original coefficient of friction was restored. A summary of the results is as follows:

Description	Before cleaning	After cleaning
Million gallons daily	0.80	1.18
Total pressure (lb per sq in.)	141	122
C in the Hazen-Williams formula	76	132

The static head is 162 ft.

The concentration of solids in the sludge varied from 1.6% in 1938 to 3.7% in 1941. An examination of the line during the cleaning process showed a thin layer of very fine grit on the bottom segment of the pipe, fairly clean sides, and considerable grease and other separated material hanging from the top. Air-relief valves were not installed on the Cleveland line.

Effluent Filtration

Magnetite filters have revived an interest in effluent filtration. Earlier tests at Chicago, about 1914, with a rapid sand filter on settled effluent indicated difficulty in handling the wash water and in keeping the filter from becoming septic. At that time the use of chlorine was in its infancy. During World War I, settled effluent was treated on a rapid filter at the Great Lakes Naval Training Station in Illinois. The filter became clogged. The handling of the wash water was also unsatisfactory.

The first installation of a magnetite filter was at Dearborn, Mich., by W. C. Laughlin, in 1932, in a radial-flow, circular, clarifier tank for treating the effluent of a chemical precipitation. Later tests were made at Chicago on the filtration of activated sludge and other effluents and at Coney Island, in New York City, and Perth Amboy, N. J. (56)(57). At present there are approximately twenty-three installations in the United States, the largest being at Minneapolis-St. Paul with 134 mgd. Such filters have been installed to follow activated sludge, high-rate trickling filters, chemical precipitation, and plain sedimentation. A few installations are treating industrial wastes. The present trend in design is toward a down-flow circular filter, with a finer magnetite sand than was formerly used. There are also rapid sand filters in service at Somerville, Raritan, Sayreville, South River, and Rutherford, N. J.

The choice of such effluent filters is determined by the operating conditions and the economies of alternative methods. At the North Side Works in Chicago it appeared better to provide additional final settling tanks at less cost. There is need of data on performance and operating and maintenance costs. At present the field of effluent filtration seems quiet.

Odor Control from Drying

Consideration of the control of odors is essential where sludge is dried and incinerated. From the observations at Chicago, Milwaukee, and Pasadena, it is obvious that water sprays or chlorine will not remove odors. Passing the vented vapors through a temperature of approximately 1,200° F appears to be the best method. A unique method of odor control is used at Pasadena, called the Cottrell-Royster (58), in which the vapors are heated to 1,200° F in a "stove" full of crushed rock, and passed to another stove until the gases discharged reach 200° F. The flow to the stoves is then reversed.

Choice of Materials

The vast increase in war industry has removed from the market, at least temporarily, a number of materials used heretofore at sewage works, such as aluminum, bronze, nickel, chromium, copper, zinc, ferrous metals, and rubber. Temporary substitutes will be sought. A recent example of substitution is the

purchase by The Sanitary District of Chicago of approximately 14,000 sq ft of wooden drainage made of Port Orford (Ore.) cedar to replace worn-out bronze wire mesh in the drainage system of its vacuum filters. To clean fine screens, Nylon brushes are used in place of hog bristles at Milwaukee and New York City.

Failure of Metals

There is need for careful observation and report on the behavior of various metals in sewage works, when exposed to sewage and sewage gases. More knowledge is desirable on the use of aluminum, inasmuch as aluminum pipes are reported to have failed in a digestion tank after 20 months use at Iowa City (36), and the underwater aluminum in the Tow-Bro apparatus in the final settling tanks in the new plant at Milwaukee is reported to need replacing. There has also been corrosion of sludge digester parts (59) made of steel. Erosion or abrasion by grit on pipes and conveyers has been noted at the Calumet plant (Chicago), Buffalo (60), and elsewhere.

Paints

The difficulty of obtaining suitable materials for paint in sewage works is increasing as a result of the war stress. Shortages of many ingredients have arisen where good substitutes are not available. In many cases temporary protection can be provided for steel work, in the hope that, before the paint has failed, the war will be over. W. T. McClenahan (61) has brought up to date his original paint specifications (62). Those interested in paints for sewage works will find much information in the *Manual of Standard Paints and Painting*, issued by The Sanitary District of Chicago in 1936.

Acoustic Control

At Gary, Ind. (63), the first use of acoustical tile in a sewage works is recorded. This material lines the inside of the roof over the blower room.

Safety Measures

Another definite trend is the provision of safety measures during the construction of large intercepting sewers built at Chicago in tunnel under compressed air, and also in the operation of sewage treatment works and pumping stations. Modern design provides for adequate ventilation in all locations where dangerous gases might accumulate or where a deficiency of oxygen might occur (64)(65).

Industrial Waste

Interest in the treatment of industrial wastes continues. With the sudden and rapid growth of war industries, many localities face special problems. However, federal and state authorities are cooperating toward the solution of these problems.

In the iron and steel industry an active effort has been made by W. W. Hodge (66) for more than 2 years to find the best way to handle spent acid pickling liquor. Mr. Rudolfs (67)(68) has listed certain chemicals which are

toxic to the digestion of sludge, such as copper sulfate, sodium arsenate, and gasoline. The Committee on Sewage Disposal (7) has listed the wastes known to be toxic to activated sludge.

Air Conditioning

The growth of air conditioning in a large city is illustrated by the experience in Chicago between 1932 and 1940. In the entire city the number of installations has risen from 255 to 4,846 and the tonnage of refrigeration from 18,000 to 66,562. If the use of water is assumed to be 2 gal per min per ton of refrigeration, in 1940 the total installation (66,562 tons) in Chicago required 133,124 gal per min, or 192 mgd, of which 39,466 gal per min, or 57 mgd (19,733 tons), originated in the down-town (Loop) area. The load factor, however, is somewhat uncertain but was estimated by the city engineer at 58% in 1938 and 70% in 1939 and 1940. About three tenths of the tonnage is installed in the down-town area bounded by the Chicago River on the north and west, Lake Michigan on the east, and Roosevelt Road (12th Street) on the south. Thus, at the peak of warm weather, the air-conditioning flow may be between 10% and 15% of the total sewage flow.

Relation of Garbage to Sewage Disposal

The handling of garbage at sewage plants is now on trial at Rock Island, Ill., Lansing, and Gary. In all these places the primary sludge is digested. The garbage is stated to be hauled to the plant, where it is crushed. Trouble is reported with bones, bottle caps, etc., at Lansing. At Gary, legal interference has prevented the treatment of garbage. Ground garbage mixed with raw sedimentation or chemically precipitated sludge is being dried and incinerated at Kaukauna, Wis., Conshohocken, Pa., and Ashland, Ohio. At the latter plant, liquid sludge is handled directly, without dewatering.

Obviously, for handling in a sewage plant, garbage should be thoroughly and completely separated from rubbish and other refuse, both combustible and incombustible. The relation of the dry weight of garbage to the dry weight of sewage solids per capita is approximately as shown in Table 4.

TABLE 4.—Relation of Dry Weight of Garbage to Dry Weight of Sewage Solids

Material	Range of Dry Solids, in Lb per Capita per Yr		Solids on Dry Basis (%)			
			Volatile		Ether Soluble	
	From:	To:	From:	To:	From:	To:
Garbage..........	30	100	60	88	11	20
Sewage Solids:						
Combined sewers	60	192	45	85	....	....
Separate sewers	40	118	50	95	....	....

After the solids have been treated, whether by the activated sludge process, by digestion, or otherwise, the content of volatile and ether-soluble matter is usually altered. The grease content of garbage is commonly estimated as higher than that of sewage solids or sludge.

Garbage contains from 70% to 80% moisture. Sewage solids, according to the type and amount of treatment, contain from 85% to 99.5% moisture before

being dewatered. In cases where the sewage plant is of the activated sludge type, with dewatering and incineration of solids, or drying for use as fertilizer, the procedure of grinding the garbage and dumping it into a sewer or sewage works, and then dewatering the mixture of garbage and excess activated sludge to a cake of approximately 80% to 85% moisture, is of doubtful value. Obviously, the cost of such treatment may easily exceed the cost of hauling the garbage to the site. However, it may prove economical to handle both sewage and garbage at a common site, but in separate works, as Indianapolis has done for many years.

Effect of Garbage on Plant Load

At Findlay, Ohio (69), both the biochemical oxygen demand (B.O.D.) and suspended solids increased markedly when ground garbage was added to the sewage (Table 5). The grit was quadrupled.

TABLE 5.—Effect of Adding Ground Garbage to Sewage

Material	Parts per Million		Population equivalent
	5-day B.O.D.	Suspended solids	
Sewage	182	149	18,080
Sewage plus garbage	278	301	25,743
Percentage increase	53	101	42

Digestion of Mixed Garbage and Sewage Sludge

The biologic digestion of garbage with sewage sludge was investigated (70) at the University of Illinois, at Urbana, in tanks 10 ft in diameter. The tests showed that ground garbage could be fed into the sewage influent of an Imhoff tank up to 1½ tons per million gal. Sludge accumulated at the rate of 2 cu yd per million gal of sewage per ton of garbage added, when retained for 1 year in the digestion compartment. Ground garbage, well mixed with sludge, was fed to a heat-controlled, two-stage, separate digestion tank, with constant and rapid recirculation of the digesting mixture at 90° F. Equal weights of garbage and sewage volatile solids were fed at the rate of 1.67 lb of volatile solids per day per cubic yard of digestion capacity. The period of detention was 30 days. Such a rate of feeding calls for a digestion capacity of 3.75 cu ft per capita on the basis of 0.115 lb of garbage volatile solids and an equal weight of sewage volatile solids per day per capita. The capacity of the tank was not reached. The temperature-controlled digesters could be operated at a loading of 3.45 cu ft of digester capacity per capita when fed with elutriated garbage mixed with fresh sludge, and at a loading of 2.88 cu ft per capita when fed with non-elutriated garbage mixed with fresh sludge. These figures are based on a detention period of 30 days. The normal rate of operation was 0.06 lb of volatile solids per day per cubic foot of tank capacity. A gas production of 13 cu ft per lb of volatile solids was attained. At Goshen, N. Y. (71), a digestion space of 4 cu ft per capita is provided with heat control.

Poliomyelitis

One of the more recent developments in public health as it relates to sanitary engineering is the suggestion that sewage may carry the virus of poliomyelitis.

J. R. Paul, M.D., and the late J. D. Trask, M.D. (72), claim that the virus has been found repeatedly in sewage but not in water. In their opinion, the virus is more resistant than B. coli or B. typhosus. Laboratory studies show that it may survive in water and milk for long periods but is killed by heat. The effect of chlorine is uncertain from limited studies. Doctors Paul and Trask conclude that it is questionable whether the presence of virus in sewage provides epidemiological evidence. Other observers claim that flies or mosquitos may carry the disease.

Whether poliomyelitis will be classed as a water-borne disease is now subject to scrutiny. Further study is needed to determine whether it may be a danger to bathers in sewage-polluted waters or whether it may be transmitted by shell fish.

Litigation

The most recent litigation in the field of sanitation was the petition of the State of Illinois to the Supreme Court of the United States for a reopening of the so-called "Lake Level Case" to permit an increased diversion from Lake Michigan at Chicago for 2 years. A special Master was appointed who made a report, upon which the court denied the petition. The case offers a novel topic concerning what constitutes a menace to health.

Patents

The last of the Jones patents in the United States on the activated sludge apparatus and process, which were in litigation, expired on November 25, 1935. Therefore, it is now possible to design and operate an activated sludge plant without infringing on any of the Jones patents.

A considerable number of patents are now being issued yearly on sewage treatment apparatus and processes, which are worked but little, if at all. Seemingly, the profession would be benefited if such patents were dedicated to the public, and thereby further research would be stimulated.

From the number of patents issued yearly in the United States on sewage apparatus and procedure, it would appear that the patenting of devices has outstripped the testing of the value of such devices. In other words there are many patents which are not backed by an adequate knowledge of performance.

Societies

For many years there have been at least three major groups, each providing an opportunity for the discussion of matters relating to sewage treatment:

The Society is largely concerned with the design and the construction of structures.

The American Public Health Association is largely concerned with general sanitary problems as they affect the health or comfort of the people.

The Federation of Sewage Works Associations is largely concerned with the research in sewage treatment and the operation of sewage works. Originally a loosely knit organization of local groups with local meetings, it became a national organization in 1940.

The Committee feels that these groups are foremost in the field and are entitled to the support of the engineers engaged in sewage treatment works who are qualified for the respective memberships.

With regard to laboratory procedure, the American Chemical Society has a division which covers the chemical field, cooperating with the Laboratory Section of the American Public Health Association.

The Committee believes that, from time to time, these groups should prepare manuals of practice on sewage works operation and control, similar to the Manual of Water Filtration Practice or the Manual of Water Works Practice, but covering a wider field.

Respectfully submitted,
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APPENDIX

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

SEWER RENTAL LAWS AND PROCEDURE

THIRD PROGRESS REPORT
OF THE
COMMITTEE OF THE SANITARY ENGINEERING DIVISION

The Committee was formed in 1935 and a progress report published in 1937.[1] In 1940 a partial progress report was prepared and presented at the January, 1941, meeting, but was not published. For the 1941 report the Committee has obtained additional replies to requests for information which is included in this report (see Appendix 1).

It was hoped that a meeting of the Committee could be held this year and arrangements were made for the necessary travel funds and expenses. Because of the increase in national defense activities, however, it has been impossible to arrange for such a meeting to formulate suggested legislation.

As stated in the second (unpublished) progress report for 1940, no meetings have been held since the formation of the Committee, although there has been considerable correspondence and occasional conferences with individual members. Beginning early in 1940, however, the Committee undertook the preparation of questionnaire forms (see Fig. 1) to aid in the compilation of state laws and essential data relative to the use of sewer rental procedures on a nation-wide basis. This has entailed an exchange of correspondence of considerable magnitude.

To facilitate the collection of information, a geographical division of the United States, by states, was made, assigning to the Committee members those states in the general vicinity of their residences. Considerable data concerning the laws and general information on those municipalities having financed sewer and sewage disposal construction by sewer rents have been obtained and are summarized in Appendix 1. It should be remembered that these summaries cover information on sewerage and sewage disposal construction financed by sewer rentals in the United States, with the exception of the State of Mississippi from which no reply was received. The information regarding the laws is based on replies when received. Changes may have occurred in the meantime, of which the Committee has not been advised.

NOTE.—This Progress Report was presented at a meeting of the Sanitary Engineering Division, New York, N. Y., on January 21, 1942. Please forward all comments on this Report directly to Chairman Earl Devendorf, Asst. Director, Div. of Sanitation, State Health Dept., Albany, N. Y.

[1] *Civil Engineering*, March, 1937, p. 220.

QUESTIONNAIRE RE FINANCING OF SEWAGE WORKS

Gentlemen:

During recent years many municipalities have financed the construction of sewers and sewage treatment by means of sewer rentals or by revenue bonds. To obtain data on such projects that will be helpful to the engineering profession and municipal administrative officials, the American Society of Civil Engineers appointed a Committee on Sewer Rental Laws and Procedure to compile a digest of the various state laws and procedures under which this method of financing the cost of construction and maintenance of sewer systems and sewage treatment plants have been undertaken.

Information submitted to our committee indicates that your community has undertaken the construction of sanitary sewers or sewage treatment works financed by sewer rentals or revenue bonds.

To assist the committee in compiling these data it will be very helpful if you will complete the attached questionnaire in triplicate and return two copies to me at your earliest convenience.

Your cooperation in furnishing this information will be greatly appreciated.

Very truly yours,
Member,
Committee on Sewer Rental Laws and Procedure,
American Society of Civil Engineers

1. Name of Place:..........
   (Note whether city, village or district)
2. Population served:..........
3. Date of Ordinance (Furnish 3 copies):..........
4. Basis of Charge:
   Water Consumption:..........
   Flat Rate:
   a. Number of Connections:..........
   b. Type and number of fixtures:..........
   Other basis:..........
5. Are sewer rental funds used for maintenance and operation charges for:
   a. Combined or sanitary sewers..........
   b. Intercepting sewers..........
   c. Sewage treatment plant..........
6. Are sewer rental funds also used (entirely or partly) for amortization and interest charges for:
   a. Combined or sanitary sewers..........
   b. Intercepting sewers..........
   c. Sewage treatment plant..........
7. Bills rendered:
   a. By what agency..........
   b. In what manner..........
   c. How frequent..........
8. Sewer rental charges:
   a. Minimum charge:..........
   b. Rate schedule:..........

FIG. 1.—FORM OF QUESTIONNAIRE

Table 1 is a summary of information on the amount of sewerage and sewage disposal construction completed and financed by sewer rents in connection with the Federal Aid program during recent years. This information was obtained

TABLE 1.—PARTIAL LIST OF NON-FEDERAL SANITARY SEWER PROJECTS AND SEWAGE DISPOSAL PLANT PROJECTS OF THE PWA FINANCED BY REVENUE BONDS

State	Number	Loan	Grant	Total
Alabama	23	$ 846,680	$ 499,116	$ 1,345,796
Arizona	4	450,500	190,625	641,125
Arkansas	36	1,834,100	1,281,810	3,115,910
Colorado	1	17,000	13,909	30,909
Florida	6	474,000	386,489	860,489
Illinois	14	1,002,500	500,410	1,502,910
Indiana	12	2,116,400	945,801	3,062,201
Iowa	3	386,800	145,065	531,865
Kentucky	6	562,400	352,837	915,237
Louisiana	4	142,500	116,589	259,089
Michigan	9	13,435,754	10,434,722	23,870,476
Mississippi	2	56,000	45,817	101,817
Nebraska	1	35,880	14,006	49,886
New Mexico	3	96,782	59,873	156,655
New York	2	3,545,000	6,903,000	10,448,000
North Dakota	1	4,000	1,515	5,515
Ohio	4	1,106,664	610,281	1,716,945
Oregon	6	283,500	158,677	442,177
South Carolina	12	1,403,051	503,960	1,907,011
Tennessee	13	775,600	381,098	1,156,698
Texas	46	2,420,056	1,130,936	3,550,992
Utah	2	59,900	25,695	85,595
Washington	4	87,432	64,292	151,824
West Virginia	13	899,500	600,920	1,500,420
Wisconsin	1	35,000	43,000	78,000
Puerto Rico	2	354,000	134,636	488,636
Total	230	$32,430,999	$25,545,079	$57,976,078

through the courtesy of Elmer W. Clark, M. Am. Soc. C. E., formerly Administrator of the Public Works Administration (PWA).[2]

References relating to the financing of sewerage and sewage treatment construction by sewer rentals are given in Appendix 2.

A brief review of state laws indicates that, in general, they are similar. Time has not permitted a detailed examination of the laws sufficient to prepare any suggestions for recommended legislation of this type, and this is too important to be undertaken without careful consideration and a full meeting of the Committee. Before any such recommendations for legislation can be made, it is desirable also to make a careful review of any legal decisions on sewer rental procedures that may be available.

The Committee would appreciate any suggestions, corrections, or additions that may appear desirable.

It is believed that the information in this report is the most complete thus far assembled and that it will afford an opportunity for sanitary engineers to review the large amount of material assembled. It is hoped that this will

[2] PWA List No. SP-1296, December 12, 1939.

stimulate discussion and correspondence which will be helpful in preparation of a final report of the Committee.

Respectfully submitted,

Howard R. Green — Leon B. Reynolds
W. D. Hatfield — Frederick H. Waring
Francis W. Herring — James A. Tobey, *Advisor*

Earl Devendorf, *Chairman*
Committee on Sewer Rental Laws and Procedure.

January 21, 1942

APPENDIX 1

Summary of Replies to Questionnaire

Table 2, Appendix 1, contains information concerning the number of states having sewer rental legislation, the number of municipalities employing sewer rentals, and general information on the state legislation under which such sewer rental construction has been authorized and built. From this table it will be seen that there are 27 states having such legislation and 676 municipalities which thus far have completed construction under sewer rentals.

TABLE 2.—Summary of State Laws on Sewer Rentals

Alabama (Legislation enacted; 27 cities have sewer rentals)
Alabama General Laws, Special Session 1933, Act No. 102, authorizes any county, city, or incorporated town of the state to purchase or construct a water-works system, water supply system, sewer system, sanitary disposal equipment and appliances and authorizes any county, city, or incoporated town of the state to improve, enlarge, extend, or repair the same; and for any such purpose or purposes authorizes any such county, city, or incorporated town to issue revenue bonds payable solely from the revenues derived from the operation of any such system or systems. Act 154, Regular Session 1935, authorizes any county, incorporated city, or town in the state to acquire by gift or purchase, to construct, reconstruct, improve, better, or extend and maintain and operate sewer systems and authorizes the issuance of revenue anticipation bonds payable solely from the revenues of such undertakings to finance the same. Three copies of Act No. 102 and one copy of Act 154 were received.
Arkansas (Legislation enacted; 35 cities have sewer rentals)
State legislation enacted for financing construction and maintaining sanitary sewers and treatment works with sewer rentals. No further data available.
Arizona (No legislation; 5 cities have sewer rentals)
From 1934 to 1939 Arizona had a law legalizing sewer rentals as a measure of financing improvements by means of revenue bonds which could only be procured through the federal government. This power was not renewed by legislature and the law went out of effect December 31, 1939. Five cities had taken advantage of that law, and continue to charge rentals but no more can adopt this method until further legislation, which is anticipated within the next few years. Two copies of Revenue Bond Act of 1934 and two copies of Amendment, 1939, were received.
California (Legislation enacted)
California has adopted a Sewer Revenue Bonds Act of 1939. Under this act any portion of a county, city, and country, or of any municipality incorporated under the laws of the state, may be formed into a sewer district for the purpose of acquiring or constructing any system, and may issue bonds of said district payable solely from revenues of such system. The only case known where a city used a sewer rental ordinance is Madera, many years ago, and long since ruled out by court action. Places having Public Utility Districts may make charges like any utility. There are six such districts, two of which have not begun to operate. Two copies of Sewer District Revenue Bond Act, 1939, were received.
Colorado (Legislation enacted)
Sewer Rental Law of Colorado effective June 7, 1937, permits the governing body of any city, town, or incorporated sewer or sanitary district to fix, by ordinance, such rates and charges for connections with

TABLE 2.—(*Continued*)

COLORADO (*continued*)

and use of, the sewer or sewerage systems of said municipalities or districts as may be just, reasonable, and necessary, and provide the manner of levying and collecting such rates and charges. It further provides that the revenue for such shall be placed in a separate fund to be used for operation and maintenance, any surplus to be placed in a sinking fund for renewals or extensions or for retiring the bonded indebtedness. Organization of Water and Sanitation Districts Law effective March 25, 1939, provides for the manner of organizing sewer districts. Two copies of each of the aforementioned laws were received.

CONNECTICUT (Legislation enacted)

Sections 128*c* to 150*c*, entitled Sewerage Systems in Municipalities, authorize the issuance of revenue bonds and establishment of service charge or sewer rental charge. Copies were received.

DELAWARE (No legislation; 1 city has sewer rental)

IDAHO (No legislation)

Model bill proposed to 1936 legislature passed both houses, was signed by governor, but was declared unconstitutional by supreme court.

ILLINOIS (Legislation enacted; 72 cities have sewer rentals)

Sewerage Revenue Bond and Service Charge Enacted in 1933 and amended in 1934: An Act authorizing cities, villages, incorporated towns and sanitary districts having a population of less than 500,000 to construct or acquire, improve and extend a sewerage system and impose and collect charges and rates for use thereof, issue revenue bonds, payable solely from revenue derived from operation of such system. Three copies were received.

IOWA (Legislation enacted; 13 cities have sewer rentals)

Chapter 308.2 Code of Iowa, 1939, entitled Sewer Rentals, authorizes the city or town council of any city or town which has installed or is installing sewerage, a system of sewerage, sewage pumping stations, or sewage treatment or purification works, by ordinance to establish just and equitable rates or charges or rentals to be paid to such city or town for the use of such sanitary utilities by every person, firm or corporation whose premises are served by a connection to such sanitary utilities, directly or indirectly. Net revenues may be used to pay operating costs, interest upon, and retire the principal of bonds of general obligation nature issued by vote of city or town councils. Also revenues may be pledged as a basis for issuing revenue bonds which are not otherwise a debt of the municipality. A copy of act was received.

KANSAS (Legislation enacted)

Special legislation was passed for the benefit of two municipalities as follows: Cities of the third class shall have power to provide for one or more systems of disposal works or additions to systems already built, and to build, operate, and maintain such disposal works as the governing body may designate. The cost and expense of building same shall be borne by the city as a whole, and may be paid out of the general revenue fund; or if the governing body so determines, improvement bonds of the city may be issued therefor and the proceeds from the sale of such bonds to be used in paying for the same, including engineering expense prior to construction, for which bonds may be issued.

The law was passed in one case because of a privately owned system which was purchased and continued in operation by the city on a sewer rental basis. In the other case to provide a means of collecting sewer rentals within and outside of the city because of the existence of packing houses outside the city limits. Copy of the law was received.

KENTUCKY (Legislation enacted; 9 cities have sewer rentals)

Chapter 133 of the Acts of 1926 provides that cities of the second, third, and fourth classes are authorized and empowered to purchase, establish, erect, maintain, and operate water-works and / or sewerage systems and for the purpose of defraying the cost of acquiring any such system, the city may borrow money and issue bonds, such bonds to be payable solely from the revenue funds derived from such works, the board of commissioners of each city by ordinance to set aside and pledge the income and revenue of such water works into a separate and special fund to be used and applied in payment of the cost thereof and in the maintenance, operation, and depreciation thereof. Three copies were received.

In June, 1940, a bill was passed authorizing the establishment of sanitary districts in counties containing cities of the first, second, and third classes to be known as a "sewer district," the board of directors of such district to determine rates and compensation or rentals to be charged for use of such sanitary sewers and issue bonds. A copy was received.

LOUISIANA (Legislation enacted; 5 cities have sewer rentals)

Act of Legislature of the State of Louisiana No. 222 of 1934 authorizes the formation of sewer districts. Act 31 of 1934 authorizes municipal corporations in the state to own, purchase, acquire, construct, extend, etc., sewerage systems, sewerage collection systems, sewerage treatment plants, intercepting sewers, outfall sewers, force mains, pumping stations and any and all appurtenances necessary for collection and disposal in a sanitary manner; to authorize municipal corporations to issue bonds to pay cost of such public improvements and property; to authorize municipal corporations, through their respective governing authorities, to establish, maintain and collect charges, rates or connection charges for the use and service rendered.

Act 244 of 1936 amends and reenacts Section 5 of Act of 1924 and provides that any parish in the state through its governing authority shall have authority and power and is hereby authorized and empowered by an ordinance or resolution duly adopted to establish, maintain, and collect rates, charges or connection charges for use of any service rendered by such public utility. Three copies of each were received.

TABLE 2.—(*Continued*)

MAINE (Legislation enacted; 3 cities have sewer rentals)

Chapter 251 is entitled "An Act Enabling cities and towns to take advantage of Reconstruction Finance Corporation Loans for construction of sewerage works." However, generally speaking, in the State of Maine when it is desired to establish means by which sewer rentals can be collected for maintenance or operation of sewerage systems or treatment plants, a sewer district is formed and a charter granted to it as a special legislative enactment which becomes on adoption, a private and special law. The Legislature in such act grants the right of such districts to collect sewer rentals. Three copies were received.

MASSACHUSETTS (Legislation enacted)

Chapter III of the General Laws relates to sewerage and sewage disposal, and authorizes financing or construction of sewer systems. In the state, charges are based on either metered water consumption or flat rate basis.

MINNESOTA (Legislation enacted; 7 cities have sewer rentals)

Chapter 221 of the Laws of 1935 authorizes sewer rental charges to apply to all municipalities except those of the second class organized under a home-rule charter. Special provisions for sewer rentals have also been made in the Act which created the Minneapolis-St. Paul Sanitary District. Copies of both were received.

MISSOURI (No legislation)

No enabling legislation authorizing or permitting construction or maintenance of sanitary sewers and/or sewage treatment works by means of revenue bonds or sewer rentals. No cities financing construction or maintenance of sewerage systems by means of sewer rentals.

MONTANA (Legislation enacted)

An act to Amend Section 5239 of the Revised Codes of Montana of 1931, relating to sewer systems in cities and towns and the collection of revenue to defray the cost and maintenance thereof and providing for a system of sewer rentals, states that a sewer system may be established in a city or town, which system may be divided into public, district, and private sewers and, to defray the cost of such public sewers, the city or town council may appropriate moneys therefor from the General or Sewer Fund; or by availing itself of moneys derived from a bond issue authorized by constitution and laws of the state. It further provides that when a public or main sewer also serves as a district sewer, the city council may assess the property bordering or abutting upon such public sewer, either at the time of its construction or at any future time, for an amount equal to the estimated cost of such district sewer capable of accommodating such property and the provision of such a sewer fund. To provide for the retirement of such bonds, the city council may establish and collect rentals for the use of such sewer system and may fix scale of such rentals. So far as could be learned, the Town of Plentywood does collect a sewer rental but no information could be secured regarding same.

NEBRASKA (Legislation enacted; 2 cities have sewer rentals)

Compiled Statutes, Supplement 1939, Section 18-1403 of the State of Nebraska, provides that the governing body of such municipality may make all necessary rules and regulations governing the use, operation, and control thereof. The governing body may establish just and equitable rates or charges to be paid to it for the use of such disposal plant and sewerage system by each person, firm, or corporation whose premises are served thereby. A copy was received.

NEVADA (No legislation)

There are no provisions in Nevada law for financing sewerage by sewer rentals. Financing methods are by surplus funds built up by tax assessment or bond issues made on the basis of providing a specific service, then being repaid from a yearly tax for the specific bond issue. If a rental charge is made for the service, such charge usually pays for maintenance only. There is an Improvement District Act, however, which provides for bond issues under certain conditions to provide sewerage, etc. At Elko, Winnemucca, and Lovelock a bond issue has provided funds for sewage treatment which sum was repaid by provision in tax assessment roll. Whether or not sewer rentals were charged, such funds went into the general budget and might have been used on sewerage maintenance and extensions, although the sewerage rental was not provided for that purpose nor is it recognized as a system of financing such service. Reno and Sparks repay their bonded indebtedness for improvements by provision on the tax assessment roll alone, making no charge for service but taxing for service. There are several towns charging for sewer service and such service charge is paid by the city council or county commissioners sitting as a town board.

NEW HAMPSHIRE (Legislation enacted; 1 city has sewer rental)

In 1933 the Legislature enacted, as a method of financing a particular sewage treatment plant, a law, permitting the use of a sewer rental system. This, however, applies only to "newly constructed" sewer systems and plants. In 1941 an effort was made to broaden this law to apply to current construction but the bill was rejected. In its place a law was enacted especially and specifically permitting this for the cities of Concord, Laconia, and Portsmouth only, which places desired such legislation, while a number of other municipalities were vigorously opposed, even to the adoption of a merely permissive act. The only municipality using this sytem at present is Wolfeboro, but the city councils of Concord and Laconia are now striving for it.

NEW JERSEY (Legislation enacted)

Circular 213 dated August, 1938, State of New Jersey Department of Health, revised statutes relating to waters, water supplies, and sewerage systems; Section 40: 63-7, control, maintenance, and regulation;

TABLE 2.—(*Continued*)

NEW JERSEY (*continued*)

rents and charges. The governing body shall have entire control and management of all sewers and drains therein, owned or controlled by such body, and from time to time may enlarge, increase, extend, renew, alter, replace, repair, cleanse, equip, operate, and maintain any and all sewers and drains and all other such works and structures mentioned in Section 40: 63–1 of this title, owned, or controlled by the municipality. It may, by ordinance, fix and prescribe such charges, rents, rules, regulations, conditions, and restrictions as to connection with and use of sewers and drains in the municipality. Under Article 3 of this same Circular 213 is defined the scope and definitions of self-liquidating sewers and disposal plants, authorization of projects and charges therefor. Senate Bill 84, passed by Senate and Assembly and forwarded on April 16, 1940, to the Governor provides for the formation of sewer district, the issuance of bonds to cover cost of construction and manner in which such bonds and operating expenses shall be met. One copy of Senate bill 84 and two copies of Circular 213 were received.

NEW MEXICO (Legislation enacted)

There has been a law on the books for many years which permits the charging of a fee which might be termed a sewer rental. The Village of Deming has utilized this for some time to raise funds for maintenance and operation of their system. Some attorneys have expressed the opinion that this particular law would not be held valid. In addition to Deming, which collects sewer rental fees for maintenance, the towns of Belen, Silver City, and Clovis have constructed sewers and treatment plants on what might be termed a rental basis. The Village of Lovington has a private sewerage system which collects a rental fee. The 1937 Revenue Bond Act amended in 1939, makes it possible for a city to secure bonds by the income from another utility and use the proceeds of bond sale for needed work on the other utility. Copies of the law were not available.

NEW YORK (Legislation enacted; 2 cities have sewer rentals)

In 1929 the New York State Legislature authorized the use of sewer rentals by cities and villages for financing the cost of the management, maintenance, operation, and repair of sewerage systems, including treatment and disposal works. The surplus of any such funds may be used for the enlargement or replacement of the same and for the payment of the interest on any debt incurred for the construction of such sewerage system, including sewage pumping, treatment, and disposal works and for retiring such debt, but shall not be used for the extension of the sewerage system to serve any unsewered areas or for any purpose other than one or more of the aforementioned. This legislation is embodied in Subdivision 26 of Section 20 of the General City Law and Section 279 of the village law.

The recently revised state constitution which became effective January 1, 1939, however, provides that the amount of indebtedness that may be excluded in determining the constitutional debt limit of a municipality shall be determined annually by deducting from gross revenues received therefrom during such preceding year an amount equal to all costs during such year of operation, maintenance, repairs, and replacements, and the interest on such indebtedness, and the amount required in such years for amortization or payment of such indebtedness.

In accordance with this revised constitutional amendment the legislature has prescribed the method by which the amount of such indebtedness to be so excluded shall be determined, and requires the approval of and certificate by the state comptroller based on report of the fiscal officer of the municipality on a form prescribed by the state comptroller. This last legislation became a law on April 10, 1940, under Chapter 361 of the Laws of 1940. A copy of these statutes and the State Constitution have been received.

NORTH DAKOTA (Legislation enacted; 12 cities have sewer rentals)

Legislation has been enacted authorizing municipalities to acquire, construct, reconstruct, improve, better, and extend certain revenue-producing undertakings; to maintain and operate same, and to prescribe, revise, and collect rates, fees, tolls, and charges for the services, facilities and commodities furnished thereby, and, in anticipation of the collection of the revenues thereof, to issue bonds payable solely from such revenues; regulating the issuance of such bonds and providing their payment and for the rights of the holders thereof, and other matters necessary in the premises, and declaring an emergency.

OHIO (Legislation enacted; 94 cities have sewer rentals)

The Sewer Rental Law of Ohio was enacted by the State Legislature, effective July 30, 1923. This law, codified as Sections 3891–1 to 3891–5 of the General Code of Ohio, expressly permits municipalities of the state to establish by ordinance equitable service charges to premises served by sewerage and sewage pumping, treatment and disposal facilities. When a schedule of sewerage service charges have been established by council of a municipality, the law provides that the service director, in case of a city, or the board of trustees of public affairs, in case of a village, shall manage, conduct, and control the sewerage facilities, shall collect service charges, and may make such by-laws and regulations as may be deemed necessary for the economical and efficient management of the facilities. Such charges shall constitute a lien upon the property served and if not paid when due shall be collected in the same manner as other city and village taxes. Money received by the collection of sewerage service charges must be kept separate from the general fund of the corporation and when appropriated by council it is subject to the order of the managing agency. The sewer fund shall be used for the cost of management, maintenance, operation, and repair of the sewerage, sewage pumping, treatment and disposal facilities, and any surplus in such fund may be used for the enlargement or replacement of the same and for the payment of interest and sinking-fund charges on any debt incurred for the construction of the facilities but shall not be used for the extension of a sewerage system to serve unsewered area.

Authority is given municipalities for issuing mortgage revenue bonds by the Ohio Constitution. Sewerage facilities can be declared a utility by virtue of Section 4, Article VIII, of the Constitution adopted September 3, 1912. Under the provisions of Section 12, Article VIII, of the Constitution also adopted September 3, 1912, council of a municipality may issue revenue bonds solely by councilmanic action, no vote of the electorate being necessary. Revenues may be derived by means of the Sewer Rental Law.

TABLE 2.—(*Continued*)

OKLAHOMA (No legislation)

Several efforts have been made to secure enactment of legislation in the State of Oklahoma which would permit financing of construction of sewers and sewage treatment plants by means of sewer rentals or revenue bonds, but so far these efforts have been unsuccessful. The state laws provide that these improvements must be constructed by general taxation bonds and, therefore, there are no sewer systems or treatment plants in the state which were constructed through any sort of revenue bond or rental ordinance passed locally by any community.

OREGON (Legislation enacted)

Chapter 246 of the Laws of 1935 remedies chapter 289 of the Laws of 1933. Both of these provide that any incorporated city or town in the state may own, acquire, construct, equip, operate, and maintain, in whole or in part, sewers, sewerage systems, sewage treatment, or disposal plant or plants, intercepting sewers, outfall sewers, force mains, pumping stations or ejector stations, with all appurtenances necessary, useful or convenient for treatment and disposal of sewage, and shall have authority to acquire, by gift, grant, purchase, or condemnation, necessary lands and rights of way. The governing body shall have authority to establish just and equitable rates or charges to be paid for the use of such facilities by each person, firm or corporation whose premises are served thereby. The governing body of any such municipality may, after referring the question of acquiring and constructing such facilities to a vote and after approval thereof, authorize the issuance of, and cause to be issued, bonds of such municipality for such purposes, either general obligation, limited obligation, or self-liquidating in character in a sum not more than the amount authorized at such election. Such bonds may provide for the payment of both principal and interest thereon from service charges to be imposed by said governing body. If such service charges shall be imposed such portion thereof as may be deemed sufficient shall be set aside as a sinking fund, for payment of interest and principal. Copies were received.

PENNSYLVANIA (Legislation enacted; 34 cities have sewer rentals)

Pennsylvania has a sewer rental enabling act approved July 18, 1935, P.L. 1286, as amended by Act approved May 14, 1937, P.L. 630, also non-debt revenue enabling acts for the first and third class cities for boroughs, for first and second class townships, which provide that bonds may be issued secured by a pledge of the annual rentals or charges for the use of such sewer, sewerage system, or sewage treatment works.

RHODE ISLAND (No legislation)

SOUTH DAKOTA (No legislation)

TENNESSEE (Legislation enacted; 19 cities have sewer rentals)

First law for financing of or maintaining works through sewer rentals enacted in 1933. No copies available. In 1937 another law was passed permitting construction of sewerage systems in unincorporated areas by the creation of a sewer district to be designated as a "utility district" with power to issue bonds in anticipation of collection of revenues for construction, acquisition, reconstruction, improvements or extension of said district. Bonds payable solely from revenues. Three copies were received.

TEXAS (No legislation; 325 cities have sewer rentals)

The State of Texas has no specific legislation passed which authorizes the financing of construction and maintenance of sanitary sewers and sewage treatment works with sewer rentals. There is, however, nothing that prohibits such procedure. The following is an excerpt from "Sewerage Service Charges in Texas Cities" relative to procedure:

"There is no uniform practice in Texas cities in financing the construction, extension and maintenance of their sanitary sewer systems. However, there is a definite trend toward financing by the use of a sewerage service charge. Most of the sewer systems have been financed through the issuance of revenue bonds or warrants, with a sewerage service charge being collected for the purpose of meeting the annual debt service charges and the operating expenses. This has been particularly true where PWA aid was used. But practically all of the older systems were constructed by funds provided through the issuance of bonds or warrants based upon a property tax, with the operating expenses being met from the same source—property taxes. There are many cases falling in between these two extremes where varying amounts of the annual charges are provided in part from the ad valorem tax and in part from sewerage service charges. Even though a sewer system was originally constructed with money provided by the sale of proper tax supported bonds, Texas cities may meet the annual debt requirement through the collection of monthly sewer charges. They have the power to provide, by ordinance, for the collection of such sewerage service charges for the purpose of reimbursing the tax supported funds, either in part or in full, as local conditions may require. Likewise, it is true that such cities may pay the operating expenses out of receipts provided by sewerage service charges. Then it would seem that there are two phases of financing sewerage service. One is the problem of financing the construction of the disposal plant and system of providing funds for debt service. The other problem is the one of financing the annual operating and maintenance expenses.

"Whether either or both of these two requirements should be met by a sewerage service charge, or whether the expenses should be defrayed out of general revenues obtained through property taxation is somewhat of an open question and is sometimes debated, with sound reasons on both sides of the question. There is a heavy preponderance of sound reasons in favor of the use of service charges for operation and maintenance costs. The principal objection is that there is a tendency on the part of the individual to delay the extension of sanitary sewer service to low cost rent property in order to avoid the monthly cost of the service. This is undesirable from the standpoint of the public health of the community.

"It is not so evident as to what method of financing the construction costs of a sewerage system is most desirable and equitable. In fact, it is easily possible that local conditions for different communities will point to a different method of financing for each one. Where occupancy and development

TABLE 2.—(*Continued*)

Texas (*continued*)

characteristics are fairly uniform the method of financing is not of serious moment to secure equitable distribution of the cost burden among the various users of the system. It is true that the installation of a sewer system does improve the value of the properties served. This increase in value, however, is somewhat intangible and is not at all equivalent to the direct gains affected in the fire insurance key rate and fire protection afforded by the installation of fire hydrants and large water mains. However, there can be no valid argument that the installation of a sewer system does not benefit public health and sanitation to the whole community; and for this reason there is, of course, no excuse for the argument that a general tax levy against all the property in the town is without merit. In a new municipality where all of the utilities are being developed simultaneously as a part of the initial program, it would seem feasible and advisable to put a large part of the cost of the new sanitary sewer system and treatment plant into a general bond issue financed out of taxation against the property. Since most of our Texas cities and towns, however, are not facing such an ideal condition, and since most of the property has already been evaluated on the basis of the existence of a sewer system, it seems fairer to assess the major portion of the charges for additional capital outlay and for the operation costs against the users of the system. We are heartily in accord with the idea which involves the 'pay as you go' plan. If communities, as well as individuals, will purchase that for which they have ability to pay, such irresponsible financing and development would be avoided."

Utah (Legislation enacted; 2 cities have sewer rentals)

A law enabling the sewer rental method of financing construction of new sewer systems and various construction works is known as the Granger Act, passed by the legislature in 1933. A copy was not available.

Vermont (No legislation)

Washington (Legislation enacted)

No exact information is available covering legislation enabling use of revenue methods in financing sewage treatment. Present statutes are inadequate to provide cities with an easy method of procedure and it is proposed to submit to the legislature in January, 1941, a new or revised bill for consideration.

Wisconsin (Legislation enacted; 8 cities have sewer rentals)

Chapter 133 of the Laws of 1933 relating to collection, treatment, and disposal of municipal sewage and to sewerage service charges: Any town, village, or city may construct, acquire, or lease, extend or improve any plant and equipment within or without its corporate limits for the treatment and disposal of sewage, including intercepting sewers. It may provide that the entire capital operating and other costs and expenses be paid out of the sewerage services charges. A resolution specifying the method of payment shall be adopted by a majority of the members of the governing board. Where payment is to be made in whole or in part by the issue and sale of mortgage bonds or certificates, such payments shall be made, as provided by the governing body of the municipality. Such bonds shall not constitute a general indebtedness to the municipality and shall be secured only by property and revenue of such plant. One copy was received.

Wyoming (No legislation)

No express authorization is given for construction of sanitary sewers and treatment works and the payment of same through sewer rentals. So far as treatment works are concerned, the city probably could do this by a general tax levy or it probably would have authority to assess an amount which would be in effect a sewer rental. At least one town in the state has followed that procedure and the issuance of bonds payable purely from sewer rentals was approved by some leading bond attorneys of Denver, Colo. There was no special legislation covering revenue bonds of this character.

Table 3, Appendix 1, covers detailed information of the respective municipalities in the various states where sewer and sewage disposal construction have been financed by sewer rentals. These records indicate that only two municipalities had sewer rental financing prior to 1930 and that the majority of ordinances were adopted subsequent to 1935. Included in this table is also information as to the use of sewer rentals for financing the maintenance, operation, and amortization of the sewer systems and sewage treatment works. Details of collection of rents, submission of bills, and a brief summary of minimum rates are also included. In further explanation of the charges noted in the body of Table 3, a few typical examples of the schedule of rates employed by the various municipalities based on water consumption, type and number of fixtures, as well as occupancy, are given in connection with each state.

TABLE 3.—RECORD OF RENTAL CHARGES

No.	Municipality[a]	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]
				(a) ALABAMA			
1	Fairhope (T)	1,200	297	July 17, 1935	Number of fixtures	Partly[b]	Yes
2	Mobile (C)	71,000	...	Mar. 26 1936	Water consumption	Yes	None
3	Pell City (C)	1,000	...	Dec. 1935	Number of fixtures	...	...
4	Winfield	1,000	165	Feb. 5, 1936	Number of fixtures	Yes	Yes

From a list of approximately 27 municipalities with sewer rentals, replies were received from only the four indicated:

1. *Town of Fairhope.*—Sewer rental is used partly for maintenance and operation of sanitary sewers and for the operation of the treatment plant. It is used also for the payment of amortization and interest on sewers and treatment plant. Charges are a minimum of $1.00 per month with a 50% discount to apply only on the minimum rate unless the government requires the town to charge customers $1.00 per month as outlined in the Loan and Grant Application. Average structure with bathroom, including toilet and lavatory and kitchen sink will be subject to minimum rate. Additional fixtures will be charged at the rate of 5¢ per month. Charges to industrial concerns will be based on amount and character of sewerage for the present. Bills are rendered monthly.
2. *City of Mobile.*—Where the water used or consumed equals 3,750 gal or less a fixed charge of 30¢ per month plus 10¢ per thousand gal on all water used or consumed in excess of 1,750 gal and not exceeding 3,750 gal. Where the water consumed or used is in excess of 3,750 gal per month, the following fixed charges will prevail: 30¢ up to 10,000 gal; 50¢ for 10,000 to 25,000 gal; $1.00 for 25,000 to 50,000 gal; $2.00 for 50,000 to 100,000 gal; $2.50 for 100,000 to 500,000 gal; $5.00 for 500,000 to 1,000,000 gal; $7.50 for 1,000,000 and more and a gallonage charge of

Consumption in excess of one million gallons	Charge (cents per thousand)	Consumption in excess of one million gallons	Charge (cents per thousand)
First 5,000 gal	10	Next 50,000 gal	2
Second 5,000 gal	8	Next 400,000 gal	$1\frac{1}{2}$
Third 5,000 gal	6	Next 500,000 gal	$1\frac{1}{4}$
Next 10,000 gal	4	> 1,000,000 gal	1
Next 25,000 gal	3		

Bills are rendered monthly.
3. *Pell City.*—Minimum charge of 85¢ for four connections, with 20¢ for each additional outlet. Bills are rendered monthly.
4. *Winfield.*—Minimum charge of $1.00 for four unit connections. Bills are rendered monthly.

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge	M and O	A and I
				(b) ARKANSAS			
5	Fayetteville	8,200	...	Nov. 13, 1935	Nature of occupancy	Yes[b]	Yes[b]
6	Hughes	1,000	...	Dec. 12, 1938	Type and number of fixtures	Yes	Yes
7	Paragould	6,000	...	June 23, 1939	Water consumption	Partly	Partly
8	West Memphis	3,500[b]	...	Nov. 20, 1934	Water consumption	Yes[b]	Yes[b]
9	Harrisburg	1,195	130	Nov. 25, 1938	Type of occupancy	Yes	Yes

[a] "T" denotes "town," "C" denotes "city," "V" denotes "village," and "B" denotes "borough."
[b] Charges in given cases and an explanation of rates are given with each state section.

TABLE 3.—(*Continued*)

(*b*) ARKANSAS (*continued*)

Of approximately 35 cities with sewer rental laws, replies were received from only five:

5. *Fayetteville.*—Rental income is used for the maintenance and operation of the sewage treatment plant. In the case of amortization and interest payments it is used in the case of the trickling filter only. Rates are graduated according to nature of occupancy, as follows:

Nature of Occupancy	Rate (dollars per month)
Residence	0.50
Business office and larger users	1.00
Hotels	5.00
Veterans' hospital and university	10.00

Bills are rendered every sixty days.

6. *Hughes.*—The rate schedule for Hughes is as follows:

Class	Type of service	Rate (dollars per month)
	Domestic sewer service—	
A	In any dwelling	1.50
B	In any outhouse	1.00
	Sewer service to any business house—	
C	Not for public use (1 water closet and minor fixtures)	1.75
C	Additional water closets, each	0.25
	Where facilities are open to the public	
D	One water closet and minor fixtures	2.00
D	Additional water closets, each	0.25
	Service to schools—	
E	Each water closet	1.00
E	Maximum charge to any one school	10.00

The foregoing rates should produce a total revenue sufficient in each year for the payment of proper and reasonable expense of operation, repair, replacements, and maintenance of such projects and for the payment of sums required to be paid into a sinking fund for revenue bonds and interest thereon. The minimum charge in all cases is $1.00 and bills are rendered monthly.

7. *Paragould.*—Rental income is used to pay maintenance and operation costs of the sanitary sewers in a new addition and the disposal plant; and also the amortization and interest charges on the sanitary sewers in the new addition. Bills are rendered monthly with the water bill from the city water department. Minimum charge of 35¢ up to 2,000 gal is as follows:

Consumption (gal per month)	Rate (dollars per month)	Consumption (gal per month)	Rate (dollars per month)
< 2,000	0.35	15,000 to 20,000	1.25
2,000 to 3,000	0.40	20,000 to 30,000	1.50
3,000 to 4,000	0.45	30,000 to 50,000	1.75
4,000 to 5,000	0.50	50,000 to 100,000	2.00
5,000 to 10,000	0.75	> 100,000	2.50
10,000 to 15,000	1.00		

[c] "M and O" denote "maintenance and operation, respectively, of sanitary sewers and sewage treatment plants."

[d] "A and I" denote "amortization and interest, respectively, on sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

(*b*) Arkansas (*continued*)

Additional charge 50¢ per month from all users in territory served by extension.

8. *West Memphis.*—Only 30% of the total population of this city is served by sewers, which are both combined and sanitary. This is a city of the second class. Bills are mailed monthly by the office of the city engineer. Rates in effect January 1, 1938, are as follows:

Nature of occupancy	Rate (dollars per month)
Universal minimum	0.75
Residences having:	
No bathroom	0.75
One bathroom with three or more fixtures	1.00
More than one bathroom	1.25
Places of business, stores, etc.—	
Having no public toilets and using (in gal per month)	
< 5,000	1.50
> 5,000	2.25
Having one or two public toilets and using (in gal per month)	
< 5,000	2.50
5,000 < 35,000	3.50
35,000 < 60,000	4.50
> 60,000	6.00
Having more than two rest rooms; minimum	6.00
Schools, public or private; 10¢ per month per pupil for the daily average number of pupils; minimum	2.50
Factories, industries, etc.; $3.00 monthly, plus 10¢ per month per average daily number of employees; minimum	3.00
Apartment houses having one or more common bathrooms; minimum per bathroom	1.25
Hotels, rooming houses, tourist camps, etc.—	
Using 15,000 gal per month or less,	
Minimum rate per sleeping room	0.30
Minimum charge	2.25
Using more than 15,000 gal per month; $3.50 per month plus 10¢ per month per sleeping room; minimum	3.50
Civic centers, community houses, city halls, etc.	5.00
Neighborhood toilets and bathhouses, where one bathroom or toilet is constructed apart from regular buildings to accommodate more than one house or family, minimum charge per toilet or bathhouse for each family or business using it	0.75

9. *Harrisburg.*—Minimum rates for dwellings, $1.25 per month, and for business, $1.50 per month. Bills are rendered monthly.

[a] "T" denotes "town," "C" denotes "city," "V" denotes "village," and "B" denotes "borough."
[b] Charges in given cases and an explanation of rates are given with each state section.

TABLE 3.—(*Continued*)

No.	Municipality[a]	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]
				(c) Arizona			
	Phoenix:						
10	City	66,000	600	Mar. 10, 1938	Water consumption except dwellings at flat rate	No	No
11	Suburbs only	3,500	...	....		...	...
12	Wickenburg (T)	1,000	241	Nov. 26, 1936	Number of water fixtures	No[b]	Yes[b]
13	Williams	2,500	...	Mar. 11, 1937	Flat rate	Yes	Partly

The state has only about five places using sewer rental laws, replies to the questionnaire being received from three.

11. *Phoenix.*—The only extraordinary case is Phoenix, a city having a population of 66,000, the sewer service being extended to about 3,500 outside the city limits. Sewer rental is charged for areas outside city limits only. For hotels and institutions, the rate is 7¢ per 1,000 gal computed as 90% of metered water consumption. Dwellings are on flat rate of $36 per year, minimum, payable monthly in advance.
12. *Wickenburg.*—Income from sewer rentals is used to pay amortization and interest on sewer bonds; but general funds are used for maintenance and operation. The rate is $1.00 per month for residences and $1.50 per month for commercial use, based on the number of water fixtures. Bills are rendered monthly.
13. *Williams.*—The rate is 75¢ per month for each water user, bills being rendered monthly.

				(d) California			
14	Wasco District	3,500	...	Feb. 23, 1937	No charge[b]	No[b]	No[b]

14. *Wasco Public Utility District.*—The cost of maintenance and operation is paid from the general fund and state and county taxes are levied for the payment of sewer bond principal and interest.

				(e) Colorado			
15	Glenwood Springs	2,000	...	....	Number of fixtures	Yes[b]	Partly[b]

15. *Glenwood Springs.*—Income from rentals is used for the maintenance and operation, and for the payment of principal and interest on the sewage treatment plant. Bills are rendered quarterly on the basis of 28¢ for each fixture (minimum, $1.40 per quarter), with a gradual advance for additional fixtures.

				(f) Connecticut			
16	Darien (V)	4,500	...	1925 and 1931[b]	Type and number of fixtures	Yes	No
17	Manchester	16,000	...	None	30% of water bill	Yes	Yes

[c] "M and O" denote "maintenance and operation, respectively, of sanitary sewers and sewage treatment plants."

[d] "A and I" denote "amortization and interest, respectively, on sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]
				(*f*) Connecticut (*continued*)			

16. *Darien.*—Adopted sewer rentals under Special Acts No. 71 and No. 288, dated 1925 and 1931, respectively. Bills are rendered annually on the basis of $7.50 per year for fifteen fixtures or less, plus 50¢ per unit additional.
17. *Manchester.*—Bills are rendered quarterly (minimum, $3.00 per quarter).

(*g*) Illinois

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge[b]	M and O[c]	A and I[d]
18	Batavia	5,500	...	Oct. 15, 1934	Water consumption	Yes	Yes
19	Belvidere	8,000	2,000	Feb. 17, 1936	Water consumption	Yes[b]	Yes[b]
20	Collinsville	1,600	...	April 4, 1938	No charge	Partly[b]	Partly[b]
21	Crystal Lake	3,900	...	July 19 1938	Flat rate	Yes	Partly[b]
22	East Peoria	7,000	...	Mar. 5, 1940	Number of connections[b]	Yes	Partly
23	Elmhurst	15,000	4,100	Dec. 5, 1935	Flat rate[b]	Yes	Partly
24	Freeport	21,000	...	Sept. 14, 1937	Water consumption	Yes	Yes
25	Kewanee	8,000[b]	...	Nov. 12, 1935	Metered water consumption	Yes	Partly
26	Libertyville	4,000	...	Aug. 25, 1936	Water consumption	Yes	Partly
27	Litchfield	5,000[b]	...	Sept. 26, 1935	Water consumption	Yes	Yes
28	Marissa	1,800	225	Nov. 1, 1938	Flat rate	Yes	Yes
29	Marshall	2,500	...	Oct. 10, 1938	Water consumption	Yes	Yes
30	Monmouth	6,000	35	Sept. 4, 1934	Water consumption	Yes	Partly
31	Morrison	3,185	...	Oct. 25, 1938	No charge[b]	...	...
32	Teutopolis	816	112	Dec. 13, 1938	Type of occupany[b]	Yes	Yes

18. *Batavia.*—Rental income is used to pay maintenance and operation costs, and the bond principal and interest, on the sewage treatment plant. Bills are rendered quarterly on the basis of 25% of the water rate (minimum, 75¢ per quarter).
19. *Belvidere.*—This community has intercepting sewers and a sewage treatment plant. Bills are rendered quarterly on the basis of 75¢ per month, minimum, and a gradual advance for larger quantities.
20. *Collinsville.*—A bonded water plant is "tied in" with the sewer department, with the agreement that the earnings of the water department are to take care of bonds and interest.
21. *Crystal Lake.*—Income from rental is used to pay principal and interest on the sewage treatment plant. Bills are rendered quarterly. For residences there is a charge of $5.00 per year per family, and industrial and commercial establishments are charged 6¢ per thousand gallons consumed in excess of 10 thousand gallons per quarter.
22. *East Peoria.*—Dwellings are charged on the basis of the number of connections and industrial establishments on the type and number of fixtures. Bills are rendered quarterly on a flat rate of $1.50 per quarter.
23. *Elmhurst.*—Approximately seventy-two municipalities in the state collect sewer rentals. For the most part these charges are based on water consumption. As an example of an unusual case may be cited the City of Elmhurst. The services of the water works and sewerage system of the city are combined and known as Municipal Utility Service of the City of Elmhurst:

"The following rates and charges for the use of such Municipal Utility Service are hereby established:

[b] Charges in given cases and an explanation of rates are given with each state section.
[c] "M and O" denote "maintenance and operation, respectively, of sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

(*g*) Illinois (*continued*)

"(*a*) Charges for the Municipal Utility Service of the city of Elmhurst to each customer shall be determined by the sum of a Quantity Charge based upon the quantity of water delivered to the customer, and a service or connection charge:

"(*b*) The Quantity Charge for all classes of customers shall be determined by the measure of water delivered to a customer at the following rates: For the first 1,000 gallons per day average or less \$.30 per M gallons. For the next 4,000 gallons per day average or portion, \$.25 per M gallons. For the next 15,000 gallons per day average or portion, \$.25 per M gallons. For all over 20,000 gallons per day \$.15 per M gallons.

"(*c*) The minimum quantity charge shall be \$.75 per month. The foregoing quantities and rates, measured by gallons, may be expressed by equivalent substituted quantities and rates measured by cubic feet on the basis that 1 cubic foot equals $7\frac{1}{2}$ gallons.

"(*d*) Charges for each single family residence, or for each single family apartment having separately metered water service, or for each office, store or establishment of less than 6 people and having separately metered water service, shall be the quantity charge as provided in foregoing sub-section (*b*) plus a service charge of \$.80 per month. This rate shall not apply to taverns, gasoline stations, or other establishments having toilet or comfort stations for public use.

"(*e*) Charges for duplex family residences as the same are defined in the Zoning Ordinance of the city and two flat buildings having a single metered water service shall be the quantity charge as provided in foregoing sub-section (*b*) plus a service charge of \$1.60 per month.

"(*f*) Charges for mercantile or commercial establishments of more than 5 people and for apartment buildings, office buildings, hotels, hospitals, library buildings, places of amusement, schools, clubs, lodges, churches, taverns, gasoline stations, and railroad stations, and for industrial processing or service establishments, except as hereinafter otherwise provided, shall be the quantity charge provided in foregoing sub-section (*b*), plus a service charge equal to 50% of the quantity charge.

"(*g*) Charges for parks and for industrial customers using water that is not returned for disposal through the sewerage system of the City of Elmhurst shall be the Quantity Charge as in foregoing sub-section (*b*) provided, plus a service charge of \$2.00 per month.

"(*h*) Charges for all unmetered service connections which are to be used by industries, commercial, and manufacturing establishments, places of business, of any type, and private individuals, exclusively for fire protection, shall be a flat rate of 5¢ per month for every 1,000 cubic feet of the space so protected, payable on the bills as rendered."

In other words, single family residences are charged a flat rate; all other users are charged a percentage of their water bill. Bills are rendered monthly.

[d] "A and I" denote "amortization and interest, respectively, on sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

(*g*) ILLINOIS (*continued*)

24. *Freeport.*—Another unusual case in the State of Illinois is the City of Freeport, where the charges (billed monthly on the basis of consumption) are as follows:

Consumption (cubic feet)	Rate (Dollars per Hundred Cubic Feet per Month) City users	Suburban users
First 4,000	0.095	0.110
Next 16,000	0.048	0.054
Next 280,000	0.025	0.028
Next 100,000	0.010	0.015
All in excess of 400,000	0.005	0.009

The minimum charges, for corresponding services (based on the size of the meter or discharge pipe), are as follows:

	Rate (Dollars per Month)		
Size of meter or discharge pipe (inches) (1)	City users (2)	Suburban Users Metered (3)	Suburban Users Not metered (4)
$\frac{5}{8}$ or less	0.20	0.22	....
$\frac{3}{4}$	0.34	0.40	....
1	0.57	0.63	1.00
$1\frac{1}{2}$	1.34	1.40	2.00
2	2.48	2.80	3.50
3	4.63	5.20	6.00
4 or more	7.74	8.75	10.00

Users whose water is not metered are billed on a monthly basis as estimated by the water and sewer commission or its authorized representative, as indicated in the foregoing tabulation. The rate for sewer service for eleemosynary institutions in the City of Freeport, including the Y.M.C.A., the Y.W.C.A., the churches, St. Francis Hospital, Evangelical Deaconess Hospital, St. Vincent's Orphanage, and the King's Daughters' Children's Home, is \$0.025 per 100 cu ft of water consumer based upon meter readings. Sewer users who install deducting meters to measure the quantity of water consumed which does not enter the sanitary sewer shall be billed monthly, in addition to all other charges, the sum of \$1.00.

25. *Kewanee.*—In the area serviced by sewers the population is 8,000. The charges per quarter year for domestic users are:

Quarterly water bill (in dollars)	Corresponding sewer rent (in dollars)
< 1.51	1.00
1.51 to 2.50	1.25
2.51 to 3.75	1.50
3.76 to 5.00	1.65
5.01 to 6.25	1.85
6.26 to 7.50	1.90
7.51 to 8.75	2.00

[a] "T" denotes "town," "C" denotes "city," "V" denotes "village," and "B" denotes "borough."
[b] Charges in given cases and an explanation of rates are given with each state section.

TABLE 3.—(*Continued*)

No.	Municipality[a]	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]

(*g*) Illinois (*continued*)

When the water bill for such quarter is in excess of $8.75 the charge is 25% of the water bill.

For commercial property (which are defined as any premises upon which business is carried on for gain or profit) the quarterly rate is 30% of water bill provided that for such commercial users as are not users of city water the charge shall be based on number of employees at rate of 25¢ per employee per quarter.

For schools, hospitals, and clubs the charge is 30% of water bill. For parks, the charge per quarter is 10% of water bill; for public buildings the charge is $900 per quarter.

26. *Libertyville.*—Bills are rendered quarterly at a rate of 10¢ per thousand gallons in excess of 5,000 gal per quarter.
27. *Litchfield.*—The new sewer treatment plant in the City of Litchfield is in full operation. The city was compelled by the State Sanitary Water Board of the Department of Public Health of the State to construct this plant in order to abate the pollution of Shoal Creek by the discharge of raw sewage from the city into it. The Litchfield sewerage rates, among the lowest in the State of Illinois, are as follows:

	Monthly
For any creamery discharging sewage into the city sewer system.	$25.00
For any hospital discharging sewage into the city sewer system..	17.50
For any industry, employing more than 150 persons, discharging sewage into the city sewer system..................	17.50

For all others, including residences and buildings, discharging sewage into the city sewer system, 25¢ per month plus an additional charge of 5¢ per 1,000 gal or fractional part thereof of water as shown by the meter reading of the Litchfield Water Works, when the residence or building is supplied with city water. Bills are rendered monthly.
28. *Marissa.*—Bills are rendered monthly, at a rate of 80¢ per month per connection.
29. *Marshall.*—Bills are rendered monthly, on a graduated rate scale.
30. *Monmouth.*—Flat rate users are charged $2.00 per year. The minimum charge is 37¢ per month on the basis of meter readings. Bills are rendered monthly.
31. *Morrison.*—No charge for sewer rental is made. The water rate is increased to cover maintenance and operation and the retirement of revenue bonds. Bills are rendered quarterly.
32. *Teutopolis.*—Charges (billed annually) are made at a flat rate, according to the type of occupancy. The minimum charge is 75¢ per year.

(*h*) Iowa

No.	Municipality[a]	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]
33	Brooklyn (T)	1,345	...	Jan. 21, 1936	Water consumption	Yes[b]	Yes[b]
34	Cedar Rapids	56,000	14,000	Apr. 1, 1935	Water consumption	Partly[b]	Partly[b]
35	Decorah	5 000	...	Jan. 21, 1936	Water consumption	Partly[b]	Partly[b]
36	Des Moines	130,000	...	Feb. 29 1940	Water consumption	Partly[b]	No
37	Estherville	4,500	...	May 4, 1936	Water consumption	Partly[b]	Yes[b]
38	Fort Dodge	24,000	...	Feb. 19, 1938	Water consumption	Yes[b]	Yes[b]
39	Ida Grove	2,050	...	June 19, 1934	Water consumption	Yes[b]	Yes[b]
40	Mount Vernon	2,000	390	Dec. 11, 1933	Water consumption	Yes	Yes
41	Vinton	3,372	...	Dec. 24, 1936	Water consumption	...	...

[c] "M and O" denote "maintenance and operation, respectively, of sanitary sewers and sewage treatment plants."

[d] "A and I" denote "amortization and interest, respectively, on sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]

(*h*) Iowa (*continued*)

33. *Brooklyn.*—The sewage treatment plant is maintained, operated, and financed by receipts from sewer rentals. Bills are rendered quarterly with a minimum of $1.00 per quarter.
34. *Cedar Rapids.*—The first city in the United States where the problem of major industrial wastes was accepted as a municipal obligation. The city has a population of 60,000 but, due to the food manufacturing industries, has a population equivalent of almost 200,000 so far as sewage is concerned. Under Section 5 (e-1) and (e-2) of the Iowa law, numerous individual industrial contracts have been entered into. A description of the major arrangement and its development is given in an article "Cedar Rapids Agreement with the Packers" which was published in *Sewage Works Journal*, November, 1937. This article will be helpful to sanitary engineers in those communities having an acute industrial waste problem. In connection with this problem and its relation to sewage treatment, Alvin A. Appel, Industrial Waste Engineer of the Department of Public Works, in Los Angeles, Calif., has done considerable research. Only the sewage treatment plant is maintained, operated, and financed by receipts from sewer rentals. Bills are rendered monthly and bi-monthly, at 25% of the water bill, with a minimum rate of 2.5¢ per 100 cu ft.
35. *Decorah.*—Only the sewage treatment plant is maintained, operated, and financed by receipts from sewer rentals. Bills are rendered quarterly, with a minimum of 22¢ per quarter.
36. *Des Moines.*—Only the cost of maintaining and operating the sewage treatment plant is paid by receipts from sewer rentals. Bills are rendered quarterly, on the basis of 10% of water bills.
37. *Estherville.*—Only the sewage treatment plant is maintained, operated, and financed by receipts from sewer rentals. Bills are rendered quarterly, with a minimum of 25¢ per quarter.
38. *Fort Dodge.*—The sewage treatment plant is maintained, operated, and financed by receipts from sewer rentals. Bills are rendered quarterly, with a minimum of 30¢ per quarter.
39. *Ida Grove.*—The sewage treatment plant is maintained, operated, and financed by receipts from sewer rentals. Bills are rendered quarterly on the basis of $1.20 per quarter per family.
40. *Mount Vernon.*—Bills are rendered quarterly, with a minimum of 25¢ per quarter.
41. *Vinton.*—The minimum rate is 2.5¢ per 100 cu ft.

(*i*) Kansas

42	Wichita	Charges are based on contract arrangements.					

(*j*) Kentucky

43	Bowling Green	14,500	...	Jan. 6, 1933	Water consumption	Yes	Yes

[a] "T" denotes "town," "C" denotes "city," "V" denotes "village," and "B" denotes "borough."
[b] Charges in given cases and an explanation of rates are given with each state section.

TABLE 3.—(*Continued*)

No.	Municipality[a]	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]
				(*j*) Kentucky (*continued*)			

43. *Bowling Green.*—Bills are rendered monthly. There is a flat charge of 10¢ per room, plus a graduated charge for the metered water used.

(*k*) Louisiana

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge	M and O	A and I
44	Bogalusa	The rate is 50¢ per month per connection.					
45	Jennings	The rate is 50¢ per month per connection.					
46	Ponchatoula	The rate is 50¢ per month per connection.					
47	Vivian	The rate is $1.00 per month plus 25¢ per fixture.					
48	Springhill	3,000	...	Dec. 28, 1937	Water consumption[b]	Yes	Yes

48. *Springhill.*—Charges are based on the metered water readings plus a flat rate. Bills are rendered monthly, with a minimum charge of $1.50 per month, for any four fixtures or less. A charge of 50¢ per month is made for each additional commode, and for other additional fixtures the excess charge is 25¢ each per month.

(*l*) Maine

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge	M and O	A and I
49	Presque Isle	4,000	760	State	Mostly per fixture	Yes	Yes

49. *Presque Isle Sewer District.*—Bills are rendered semi-annually, with a minimum charge of $5.00 per year.

(*m*) Massachusetts

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge	M and O	A and I
50	Amherst	6,000	...	Mar. 7 1937	Flat rate[b]	Yes	Yes
51	Brockton	50,000[b]	...	1926	Water consumption	Yes	Yes
52	Marion	1,867[b]	...	Mar. 5, 1906	Per fixture	No	No
53	Natick (T)	13,850	1,716	June 9, 1894	Water consumption	Yes	Partly
54	North Adams	22,000	3,200	None	Entrance fee	No	No
55	Reading (T)	10,861[b]	...	Apr. 22, 1909	Water consumption	No	No

50. *Amherst.*—A special case in the State of Massachusetts is the Town of Amherst. This town has a population of 6,000 including students of two colleges. At a board meeting it was voted that, for the purpose of raising sufficient money annually to maintain and operate the Amherst sanitary sewer system, including treatment plant, and to repay that part of bond issue (within 10 years) as voted by town, there should be established a just and equitable annual charge to each user as follows:

By special agreement between the Selectmen and Massachusetts State College Trustees, the State College is to pay $1,000 per year for 10 years; Amherst College is to pay $600 per year; and all others are to be assessed a flat rate of $6.00 per year. For each home, hotel, fraternity house, business block, and other building of assessed valuation of more than $2,000 there is to be levied an additional sewer assessment of $1.00 per thousand assessed value up to and including a maximum of $20,000. In other words, there is a flat rate of $6.00 for any building of $2,000 or less assessed valuation plus $1.00 per $1,000 up to $20,000 assessed valuation. Bills are rendered annually.

[c] "M and O" denote "maintenance and operation, respectively, of sanitary sewers and sewage treatment plants."

[d] "A and I" denote "amortization and interest, respectively, on sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]
			(*m*) Massachusetts (*continued*)				

51. *Brockton.*—The population of Brockton is between 50,000 and 55,000. Bills are rendered quarterly at a rate of 15¢ per 100 cu ft of water consumed.
52. *Marion.*—The population of Marion is 1,867 in winter and 3,600 in summer. Bills are rendered annually at the rate of $2.50 per year per fixture.
53. *Natick.*—Bills are rendered quarterly, at 25¢ per quarter per 100 cu ft, with a graduated increase for larger quantities.
55. *Reading.*—As an explanation of the charges in the Town of Reading, the following may be noted: Charges for sewer service are computed on water consumption from water meter readings. Rate for unmetered service is $6.00 per annum; for metered service, the rates are as follows:

Consumption (cubic feet)	Rate (cents per 100 cu ft)
First 3,000	23
Next 3,000	20
Next 4,000	16
Next 10,000	12
Next 10,000	8
Next 276,000	4

All in excess of 306,000 cu ft, 2¢ per 100 cu ft

Of its total population, only about 3,000 people in Reading are served by sewers. Bills are rendered semi-monthly.

(*n*) Minnesota

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge	M and O	A and I
56	Minneapolis[b]	489 000	...	1938	Water consumption	...	Partly[b]
57	Moorhead	10,000	...	1939	Fixed by committee	Yes	Yes
58	Montevideo	5,230	...	Aug. 22, 1935	Water consumption	Yes	Yes
59	Springfield.	2,362	...	Jan. 22, 1940	Water consumption	Yes[b]	Yes[b]

56. *Minneapolis-St. Paul Sanitary District.*—In the Minneapolis-St. Paul Sanitary District each municipality passed a resolution determining the basis of rental to be charged to property within the district. In the case of Minneapolis, the following resolution was adopted:

"That the rental to be charged property within the city of Minneapolis served either directly or indirectly by the sewage disposal system constructed, maintained and operated under and pursuant to the provisions of Chapter 341, Session Laws of Minnesota for the year 1933, is hereby fixed and determined at $.028 per 100 cu. ft. for domestic meters, which said rate shall be based upon the quantity of water measured in the manner specified in that certain resolution adopted by the Minneapolis-Saint Paul Sanitary District on March 10, 1938, known as Resolution No. 435 of the resolutions of said district. The minimum sewer rental charge shall be $.37 per quarter.

"That the rental to be charged property within the city of Minneapolis is hereby fixed and determined for commercial meters as follows: First 100,000 cu. ft. per mo. $.028 per 100 cu. ft. Next

[b] Charges in given cases and an explanation of rates are given with each state section.

[c] "M and O" denote "maintenance and operation, respectively, of sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]
				(n) Minnesota (*continued*)			

200,000 cu. ft. per mo. $.015 per 100 cu. ft. Over 300,000 cu. ft. per mo. $.0075 per 100 cu. ft.

"For the purpose of paying the city of St. Paul's share of the operation and maintenance costs of the Minneapolis-Saint Paul Sanitary District sewage disposal system, a sewer rental charge is hereby levied and assessed against every lot, parcel of land, building or premises situated within the corporate limits of the city of Saint Paul, now or hereafter having any connection with the sewer system of the city or otherwise discharging domestic sewage, commercial and industrial waste, water or other liquids either directly or indirectly into the sanitary sewer system of such city, according to the size of the meter used in servicing such premises which charges shall be as follows:

Meter sizes (in inches)	Annual charges (in dollars)	Meter sizes (in inches)	Annual charges (in dollars)
$\frac{5}{8}$	2.40	3	75.00
$\frac{3}{4}$	4.00	4	180.00
1	8.00	6	360.00
$1\frac{1}{4}$	12.00	8	630.00
$1\frac{1}{2}$	18.00	10	900.00
2	36.00	12	1,350.00

"In order that property outside the corporate limits served by the sewage disposal system shall bear proportionate share of construction costs, the charge made for such property shall be double the charge herein and hereafter provided for property located within the city."

Bills to domestic users are rendered quarterly. Industrial users are billed monthly.

57. *Moorhead.*—Bills are rendered monthly. For 1,100 cu ft or less, 75¢. For all in excess of 1,100 cu ft, 6.75¢ per 100 cu ft.
58. *Montevideo.*—Bills are rendered quarterly and monthly, the minimum charge for three months being 90¢.
59. *Springfield.*—The sewage treatment plant is maintained, operated, and financed by income from sewer rentals. Bills are rendered quarterly, the minimum rate being 25¢ per month.

				(o) Nebraska			
60	Schuyler	2,808	...	July 19, 1932	Classification	...	...

60. *Schuyler.*—Bills are rendered quarterly, with a minimum charge of 25¢ per month.

				(p) Nevada			
61	Lovelock	1,200	...	....	No charge	...	...

[d] "A and I" denote "amortization and interest, respectively, on sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

No.	Municipality[a]	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used For:	
						M and O[c]	A and I[d]
			(q) New Jersey				
62	Florence (T)	4,000	700	Nov. 25, 1936	Flat rate	Yes	Yes
63	Haddonfield[b]	10,000	...	....	Type and number of fixtures	Yes	No
64	Moorestown[b]	7,000	...	1917[b]	Type and number of fixtures	Yes	Yes
65	Pemberton	600	...	....	Flat rate	Yes	Yes

In New Jersey either a flat rate or a charge according to the type and number of fixtures is made. As a typical examination the following may be quoted:

Rates for sewer connections in the Borough of Haddonfield—

		Stores
Dwelling Houses for Single Families Only		
Sink, hot or cold water or both	$2.00	$2.50 each
Each additional sink, hot or cold water or both	1.00	
Wash basin or stationary washstand	1.00	2.00 each
Each additional washbasin or stationary washstand	0.50	
Each tub, hot or cold water or both	2.00	
Each additional tub, hot or cold water or both	1.50	
Shower bath, hot or cold water or both	1.00	
Each additional shower bath, hot or cold water or both	0.50	
Water closet, pan, valve or reservoir	2.00	2.50 each
Each additional water closet, pan, valve, or reservoir	1.50	
Stationary washtubs, each tub	1.00	
Urinals	1.00	
Each additional urinal	0.50	
Fountains, each	3.00	3.00
Pump, where no sink is connected	1.00	
Water motor, each		4.00
Laundries		
Steam, drainage or wash water only	7.00	
Other laundries, drainage of wash water only	7.00	
Schools and Other Public Buildings Not Otherwise Enumerated		
Water closet, pan, valve, or reservoir	3.50	
Sink, hot or cold water or both, each	3.50	
Urinals, each	1.00	
Urinals, continuous flow	2.00	
Washstands	2.00	
Shower bath	2.00	
Unlicensed Hotels and Boarding Houses		
Kitchen sinks	5.00	
Each additional sink	3.00	
Washstand, each	1.00	
Bath tubs, hot or cold water or both	2.00	
Each additional	1.50	

[a] "T" denotes "town," "C" denotes "city," "V" denotes "village," and "B" denotes "borough."
[b] Charges in given cases and an explanation of rates are given with each state section.

TABLE 3.—(*Continued*)

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for:	
						M and O[c]	A and I[d]

(q) New Jersey (*continued*)

Water closet, pan, valve, or reservoir	2.00
Each additional	1.50
Urinals, each	2.00
Urinals, continuous flow each	3.00
Stationary washtubs, each	2.00
Barber Shops	
Each chair in use	1.00
Milk Dealer	
Sink for washing bottles	4.00
Stables	
Sink or horse troughs, each	2.50
Wash pave for carriages, each	4.00
Washstand, each	1.00
Garage for Automobiles	
Wash pave for automobiles, each	4.00
Washstand, each	1.00

62. *Florence.*—Bills are rendered monthly, $12.00 per year being the minimum charge.
63. *Haddonfield Borough.*—Bills are rendered semi-annually, a minimum charge for dwellings being $2.00.
64. *Moorestown, Township of Chester.*—The 1917 Ordinance is to be revised. At present bills are rendered semi-annually, and the minimum charge for dwellings is $11.00 per year.
65. *Pemberton.*—Bills are rendered semi-annually at $7.50 per annum per house.

(r) New Mexico

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge	M and O	A and I
66	Lovington	1,984	...	None	Flat rate	...	...
67	Silver City	5,015	...	....	Flat rate	No	Partly[b]

66. *Lovington.*—Bills are rendered monthly, the minimum rate being $2.00 per month.
67. *Silver City.*—Part of the income from sewer rentals is used to pay principal and interest charges on the sewage treatment plant. Bills are rendered annually and the rate is graduated beginning at a minimum of 75¢.

(s) New York

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge	M and O	A and I
68	Buffalo	575,000	...	1938	Water consumption	Yes	Yes
69	Plattsburgh	14,713	...	Feb. 18, 1938	Type of occupancy	Yes	Yes

68. *Buffalo.*—Bills are rendered quarterly and the rate is graduated from 40¢ per thousand cubic feet. The minimum charge is 80¢.
69. *Plattsburgh.*—Bills are rendered quarterly, the minimum charge being $6.51 per dwelling per quarter.

[c] "M and O" denote "maintenance and operation, respectively, of sanitary sewers and sewage treatment plants."

[d] "A and I" denote "amortization and interest, respectively, on sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

No.	Municipality	Popu-lation	Con-nec-tions	Date of ordinance	Basis of charge[b]	RENTAL USED FOR: M and O[c]	RENTAL USED FOR: A and I[d]
				(*t*) NORTH CAROLINA			
70	Blowing Rock	Single charge per connection; bills rendered monthly; minimum annual charge, $3.00.					
71	Carrboro	Single charge per connection; bills monthly; minimum annual charge, $6.00.					
72	Carthage	Charge according to type of connection; minimum annual, $3.00 per toilet; monthly bill.					
73	Cary	Single charge per connection; minimum annual charge, $3.00; monthly bill.					
74	Chapel Hilll	Charge according to type of connection; minimum annual charge, $12.00 per residence; monthly bill.					
75	Charlotte	Charge according to front footage; 3¢ per linear foot, minimum annual.					
76	Durham	Single charge per connection; minimum annual charge, $15,00 (surburban connections only); monthly bill.					
77	Elizabeth City	Charge based on water consumption (50% of water bill).					
78	Elizabethtown	Single charge per connection; minimum annual charge, $4.20; monthly bill.					
79	Faison	Single charge per connection; minimum annual charge, $6.00; monthly bill.					
80	Fayetteville	Single charge per connection; minimum annual charge, $4.00; monthly bill.					
81	Hickory	Charge based on water consumption; minimum annual charge, $6.00, plus 10% of water bill exceeding $5.00; suburban rates are doubled; monthly bill.					
82	Mount Airy	Single charge per connection; minimum annual charge, $7.20; in the suburbs, $10.80; monthly bill.					
83	Mount Pleasant	Single charge per connection; minimum annual charge, $6.00; monthly bill.					
84	Oxford	Single charge per connection; minimum annual charge, $12.00; commercial rate is $1.00 plus 10% of water bill; monthly bill.					
85	Pittsboro	Single charge per connection; minimum annual charge, $12.00; monthly bill.					
86	Ramseur	Single charge per connection; minimum annual charge, $6.00; monthly bill.					
87	Rowland	Single charge per connection; minimum annual charge, $6.00; monthly bill.					
88	Roxboro	Charge based on water consumption.					
89	Salisbury	Charge based on water consumption (25% of water bill).					
90	Stoneville	Single charge per connection; minimum annual charge, $6.00; monthly bill.					
91	Star	Single charge per connection; minimum annual charge, $3.00; monthly bill.					
92	Thomasville	Charge based on water consumption; minimum annual charge, $2.40 (10¢ per thousand gallons of water consumed).					
93	Tryon	Annual charge according to number of fixtures: $1.94 per faucet, $2.50 per toilet, $3.33 with bath.					
94	Whiteville	Single charge per connection; minimum annual charge, $6.00; monthly bill.					
95	Wilmington	Single charge per connection ($1.00 per toilet), quarterly bill.					
96	Winston-Salem	Charge 10% of water bill.					
97	Wrightsville	Single charge per connection; minimum annual charge, $7.50 per residence; annual bill.					
98	Granite Falls	Single charge per connection; minimum annual charge, $6.00; monthly bill.					
				(*u*) NORTH DAKOTA			
99	Dickinson	5,800	...	Dec. 19, 1938	Water consumption	No	Yes
100	Fargo	33,000	...	Mar. 2, 1936	Flat rate	Yes	Yes
101	Grand Forks	20,000	...	June 26, 1935	Water consumption	Yes	Yes
102	Hazen	900	...	....	Flat rate	No	...
103	Lisbon	1,650	...	1936	Water consumption	Yes	None
104	Mott	800	...	1928	Connection charge only	Yes	None
105	Wahpeton	4,000	...	Jan. 11 1939	Flat rate[b]	Yes	Partly[b]

99. *Dickenson.*—Minimum rate, 25¢ per month; monthly bill.
100. *Fargo.*—Residential rate, $9.00 per year; minimum commercial rate, 9¢ per thousand gallons of metered water.
101. *Grand Forks.*—Minimum rate, 50¢ per month; monthly bill.
102. *Hazen.*—Minimum rate (proposed), 75¢ per quarter; quarterly bill.
103. *Lisbon.*—Minimum rate for dwellings, $1.00 per quarter; quarterly bill.
104. *Mott.*—Bill rendered at time of installation.
105. *Wahpeton.*—Flat rate, according to type of occupancy; minimum rate, 50¢ per month; quarterly bill.

[b] Charges in given cases and an explanation of rates are given with each state section.

[c] "M and O" denote "maintenance and operation, respectively, of sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]
				(v) Ohio			
106	Andover	906	...	1926	Single charge per connection	Yes	No
107	Barberton[e]	23,934	...	1937	Water consumption	Yes	Partly
108	Bedford	6,814	...	1938	Water consumption	Yes	No
109	Bellefontaine	9,543	...	1939	Water consumption	Yes	No
110	Belleville	987	...	1938	Water consumption	Yes	No
111	Berea	5,697	...	1937	Water consumption	Yes	No
112	Bowling Green	6,688	...	1935	Type of connection	Yes	No
113	Brookville[e]	1,403	...	1938	Water use and type of connection	Yes	Partly
114	Bryan	4,689	...	1939	50% of water bill	Yes	No
115	Bucyrus	10,027	...	1939	Water use and type of connection	Yes	No
116	Burton	597	...	1928	Type of connection	Yes	No
117	Canal Winchester	906	...	1940	Type of connection	Yes	No
118	Carey	2,722	...	1939	Type of connection	Yes	No
119	Carrollton	2,286	...	1939	Single charge per connection	Yes	No
120	Celina	4,664	...	1940	Type of connection	Yes	No
121	Chardon	1,814	...	1927	Single charge per connection	Yes	No
122	Circleville	7,369	...	1939	Type of connection	Yes	No
123	Cleveland	900,429	...	1938	About 30% of water bill	Yes	No
124	Coldwater	1,787	...	1939	Single charge per connection	Yes	No
125	Columbiana	2,485	...	1929	Water consumption	Yes	No
126	Columbus	290,564	...	1937	Water consumption	Yes	No
127	Columbus Grove	1,633	...	1937	Type of connection	Yes	No
128	Convoy[e]	876	...	1938	Type of connection	Yes	Partly
129	Cuyahoga Falls[e]	19,797	...	1939	50% of water bill	Yes	Yes
130	Dayton	200,982	...	1927	Water consumption	Yes	No
131	Delaware	8,675	...	1927	Type of connection	Yes	No
132	East Cleveland	39,667	...	1938	40% of water bill	Yes	No
133	East Palestine	5,215	...	1924	Number of fixtures	Yes	No
134	Fairfield	1,240	...	1938	Type of connection	Yes	No
135	Findlay[e]	19,363	...	1933	Water consumption	Yes	Yes
136	Franklin[e]	4,491	...	1938	Water consumption	Yes	Partly
137	Gahanna[e]	417	...	1939	Type of connection	Yes	Yes
138	Geneva[e]	3,791	...	1938	Water consumption	Yes	Yes
139	Germantown	2,029	...	1938	Type of connection	Yes	No
140	Grandview	6,358	...	1932	Type of connection	Yes	No
141	Granville	1,407	...	1930	Water consumption	Yes	No
142	Greenville[e]	7,036	...	1937	Water consumption	Yes	Partly
143	Greenfield	3,871	...	1938	Water consumption	Yes	No
144	Grove City	1,546	...	1938	Type of connection	Yes	No
145	Hicksville	2,445	...	1935	Type of connection	Yes	No
146	Hiram	441	...	1925	Single charge per connection	Yes	No
147	Huron	1,699	...	1939	Water consumption	Yes	No
148	Jamestown[e]	944	...	1937	Water consumption	Yes	Yes
149	Jefferson	1,601	...	1929	Water consumption	Yes	No
150	Johnstown[e]	1,006	...	1937	Type of connection	Yes	Partly
151	Lebanon	3,222	...	1934	Type of connection	Yes	No
152	Leetonia	2,332	...	1927	Number of fixtures	Yes	No
153	Leroy	249	...	1937	Type of connection	Yes	No
154	Lewisburg	936	...	1939	Type of connection	Yes	No
155	Lima	42,287	...	1932	Water consumption	Yes	No
156	Louisville	3,130	...	1934	Water consumption	Yes	No
157	McArthur	1,188	...	1937	Type of connection	Yes	No
158	McComb	932	...	1939	Type of connection	Yes	No
159	Mansfield	33,525	...	1937	Water consumption	Yes	No
160	Marysville[e]	3,639	...	1928	Type of connection	Yes	Yes[b]
161	Massillon[e]	26,400	...	1935	Type of connection	Yes	Yes
162	Mechanicsburg	1,424	...	1937	Type of connection	Yes	No
163	Miamisburg	5,316	...	1928	Single charge per connection	Yes	No
164	Minerva	2,675	...	1928	Type of connection	Yes	No
165	New Bremen	1,485	...	1934	Type of connection	Yes	No
166	Norwalk	7,776	...	1932	Type of connection	Yes	No
167	Oak Hill	1,578	...	1937	Type of connection	Yes	No
168	Oakwood	6,494	...	1928	Water consumption	Yes	No
169	Oberlin	4,292	...	1929	Water consumption	Yes	No
170	Orrville	4,427	...	1932	Water consumption	Yes	No
171	Osborn	1,271	...	1938	Water consumption	Yes	No
172	Oxford	2,588	...	1924	Water consumption	Yes	No
173	Piqua	16,009	...	1939	Water consumption	Yes	No

[d] "A and I" denote "amortization and interest, respectively, on sanitary sewers and sewage treatment plants."

[e] Improvements financed in whole or in part by mortgage revenue bonds.

TABLE 3.—(*Continued*)

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]
				(v) Ohio (*continued*)			
174	Proctorville[e]	675	...	1934	Type of connection	Yes	Partly
175	Quaker City	613	...	1937	Type of connection	Yes	No
176	Reynoldsburg[e]	562	...	1937	Type of connection	Yes	Yes
177	Richwood	1,573	...	1938	Type of connection	Yes	No
178	Rio Grande	400	...	1938	Type of connection	Yes	No
179	St. Marys	5,433	...	1930	Single charge per connection	Yes	No
180	Sebring	3,949	...	1930	Water consumption	Yes	No
181	Shreve	1,103	...	1926	Single charge per connection	Yes	No
182	Springfield	68,763	...	1936	Water consumption	Yes	No
183	Sunbury	784	...	1940	Water consumption	Yes	No
184	Tippecanoe City	2,559	...	1940	40% of water bill	Yes	No
185	Toledo	290,718	...	1937	Water consumption	Yes	No
186	Trotwood	660	...	1939	Type of connection	Yes	No
187	Troy	8,675	...	1939	Type of connection	Yes	No
188	Union City[e]	1,305	...	1938	Water consumption	Yes	Partly
189	University Heights	2,237	...	1937	Type of connection	Yes	No
190	Upper Arlington	3,059	...	1933	Type of connection	Yes	No
191	Utica[e]	1,394	...	1938	Water consumption	Yes	Partly
192	Van Wert	8,472	...	1935	Type of connection	Yes	No
193	Wapakoneta	5,378	...	1941	Type of connection	Yes	No
194	Washington C.H.[e]	8,436	...	1934	Type of connection	Yes	Yes
195	Wauseon[e]	2,889	...	1939	Type of connection	Yes	Partly
196	West Alexandria	924	...	1939	Type of connection	Yes	No
197	Willard	4,514	...	1940	Type of connection	Yes	No
198	Wooster	10,742	...	1939	Water consumption	Yes	No
199	Yellow Springs[e]	1,472	...	1937	Water consumption	Yes	Partly

In the foregoing list of Ohio communities bills are rendered quarterly, with the relatively few exceptions noted herein. Minimum annual charges at various places are as follows:

$4.00—Andover, Barberton, Bucyrus, Burton (semi-annual bill), Carrollton, Columbus[b] (semi-annually), Dayton, Delaware, Grandview (annual bill), Granville (semi-annual bill), Grove City, Hicksville, Jefferson, Johnstown, Leetonia, Lewisburg, Louisville, Marysville (semi-annual bill), Mechanicsburg, Minerva, Quaker City, Sunbury, and Wooster;

$6.00—Bedford, Bowling Green, Bryan, Carey, Celina, Circleville, Coldwater, Columbus Grove, Convoy, Cuyahoga Falls, Fairfield, Franklin, Geneva, Germantown, Huron, Leroy, New Bremen, Norwalk, Oak Hill, Osborn, Sebring, Trotwood, Troy, Upper Arlington, Van Wert, West Alexandria, and Willard;

$2.00—Bellefontaine (annual bill), Cleveland, East Palestine, Lebanon, McArthur, McComb, Mansfield (semi-annual bill), Miamisburg (annual bill), Rio Grande, St. Marys, Springfield (semi-annual bill), and Wapakoneta;

$3.00—Belleville (semi-annual bill), Canal Winchester, Chardon, Richwood, and Shreve (annual bill);

$8.00—Berea, Hiram, Massillon, Utica, Washington C. H., and Wauseon;

$5.00—Columbiana, Greenville, Greenfield, Oakwood, Oberlin, Oxford, and University Heights;

$18.00—Reynoldsburg; $12.00—Gahanna, Jamestown; $10.00—Yellow Springs; $9.00—Proctorville; $6.65—Union City; $4.80—Findlay (monthly bill), Tippecanoe City; $4.44—Orville; $4.40—Brookville; $3.20—East Cleveland; $2.64—Piqua; $2.40—Lima; $1.50—Toledo.[b]

[a] "T" denotes "town," "C" denotes "city," "V" denotes "village," and "B" denotes "borough."

[b] Charges in given cases and an explanation of rates are given with each state section.

[c] "M and O" denote "maintenance and operation, respectively, of sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

No.	Municipality[a]	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]

(*v*) Ohio (*continued*)

126. *Columbus.*—Minimum charge of $1.66. The charge to the average water consumer approximates $4.00.
160. *Marysville.*—Improvements to the existing sewage treatment plant are supported entirely from rental receipts.
185. *Toledo.*—An 80¢ charge to the average water consumer approximates $1.50 per quarter.

(*w*) Oregon

200	Ashland	5,000	...	June 4, 1936	Flat rate	Yes	Yes
201	Burns	2,500	370	Jan. 16, 1936	Type of occupancy	Yes[b]	Yes[b]
202	Grants Pass	6,000	...	Feb. 7, 1935	Type and number of fixtures	Yes[b]	Yes[b]
203	Hillsboro[b]	4,000	...	Feb. 17, 1936	Number of fixtures	Yes	Yes
204	Medford	12,789	...	Apr. 16, 1935	Water consumption	No	Yes
205	Seaside	3,500[b]	...	Nov. 21, 1938	20% of water receipts	No	No
206	Silverton	5,000	1,250	....	Flat rate[b]	Yes	Yes
207	Talent	425	124	Jan. 25, 1936	No bills to date	...	...

200. *Ashland.*—Charges begin at a minimum of 25¢; monthly bills.
201. *Burns.*—The sewage treatment plant is maintained, operated, and financed entirely from rent receipts. Charges begin at a minimum of 75¢; monthly bills.
202. *Grants Pass.*—The sewage treatment plant is maintained, operated, and financed entirely from rent receipts. Bills are rendered monthly at a flat rate per fixture.
203. *Hillsboro, Districts 1 and 2.*—In district 1, the minimum for residences is $4.00, and for business, $6.00. In district 2, corresponding rates are $15.00 and $18.00, respectively. Bills are rendered semi-annually.
204. *Medford.*—Bills are rendered monthly and bi-monthly, on a graduated scale of rates.
205. *Seaside.*—Population varies from 3,500 in winter to 35,000 in summer.
206. *Silverton.*—Residences are charged a flat rate of 50¢ per month. Commercial establishments are charged on the basis of water consumption.
207. *Talent.*—No sewer rental bills have been rendered to date.

(*x*) Pennsylvania

208	Ambler	4,000	...	Nov. 16, 1934	Type and number of fixtures	Yes	Yes[b]
209	Cheltenham (T)	20,000[b]	...	Apr. 19, 1927	Number of fixtures	Yes[b]	Yes[b]
210	Coatesville	16,000	...	Dec. 27, 1935	Flat rate[b]	Yes	Yes
211	Haverford (T)	25,000	...	June, 1921	Type and number of fixtures	Yes	Yes
212	Jeannette	16,000	...	....	Meter readings	Yes	Yes
213	Merion (T)[b]	35,000	...	Jan. 26, 1916	Type and number of fixtures	Yes	Yes
214	Providence (T)[b]	3,900	...	Dec. 21, 1938	Type and number of fixtures	Yes	Yes
215	North East (B)	4,000	...	....	20% of water bill	Partly[b]	Yes
216	Sellersville (B)	3,000	...	....	Fixed by Public Utilities Commission	Yes	Yes
217	Souderton (B)	4,033	...	Feb. 11, 1938	Type of occupancy	Yes[b]	...
218	Swarthmore (B)	4,000	...	Mar. 1, 1939	Flat rate	Yes	Yes
219	Yeadon	8,500	...	Jan. 26, 1939	Type and number of fixtures	Yes	Yes

[d] "A and I" denote "amortization and interest, respectively, on sanitary sewers and sewage treatment plants."

[e] Improvements financed in whole or in part by mortgage revenue bonds.

TABLE 3.—(*Continued*)

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]
				(x) Pennsylvania (*continued*)			

208. *Ambler.*—Rent receipts are used for maintenance and operation. Any surplus is applied to amortization and interest charges. Bills, based on a published schedule, are rendered semi-annually.
209. *Cheltenham Township.*—Of the total population, 16,000 are served by sewers. The cost of maintaining, operating, and financing the sewerage system is paid from rent receipts. Bills are rendered annually on the basis of a schedule of fixtures.
210. *Coatesville.*—Charges are based on 60% of water bill on all metered and flat rate services. Quarterly bills are issued on meter readings, with a minimum charge of $7.20 per year.
211. *Haverford Township.*—Bills are rendered annually on the basis of the type and number of fixtures.
212. *Jeannette.*—Bills are rendered annually, the minimum charge being $7.00 for 20,000 gal of water or less.
213. *Lower Merion Township.*—For dwellings, a minimum charge of $4.00; for other buildings, a minimum charge of $5.00; annual bills.
214. *Nether Providence Township.*—Annual bills based on a schedule of type and number of fixtures.
215. *Borough of North East.*—In maintenance and operation, rent receipts are used for the sewage treatment plant only; quarterly bills.
216. *Borough of Sellersville.*—Charges, $13.50 per year per house; empty houses, $3.40 per year; semi-annual bills.
217. *Borough of Souderton.*—Rent receipts are accumulated in a general sinking fund, from which funds are drawn for maintenance and operation. Minimum charge to dwellings, $12.00 per quarter; quarterly bills.
218. *Borough of Swarthmore.*—Annual bills; minimum, $2.00 per year per dwelling.
219. *Yeadon.*—Annual bills, based on schedule of type and number of fixtures; minimum for a private house is $1.28 per year.

(y) Tennessee

220	Savannah	...	121	1936	Flat rate	Partly[b]	Yes

220. *Savannah.*—Sewer rents are not used for operating and maintaining the sewage treatment plant. Monthly bills; minimum, $1.50 per month.

(z) Texas

221	Arlington	5,000	1,188	1920	Flat rate	Yes	No
222	Ballinger	4,500	...	None	Number of fixtures	...	...
223	Brady	4,500	...	....	Type and number of fixtures	Yes	Yes
224	Canadian	2,200	...	....	Flat rate	Yes	No
225	Celeste	800	...	Dec. 11, 1936[b]	Water consumption	Yes	Yes
226	Childress	7,000	...	Apr. 23, 1918	Occupancy	Yes	Yes
227	Commerce	4,500	1,000	....	Flat rate	Yes	...
228	Corsicana	12,000	Construction in progress				
229	Crowell	2,100	318	1925	Flat rate	Yes	No
230	Dimmitt	945	240	None	Type of occupancy[b]	...	...
231	El Campo	5,000	...	No charge; supported by sinking fund			

[b] Charges in given cases and an explanation of rates are given with each state section.

[c] "M and O" denote "maintenance and operation, respectively, of sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]
				(z) Texas (*continued*)			
232	El Paso	95,000	...	May 17 1934	Water consumption	Yes	Yes
233	Fort Worth	195,000	...	Oct. 1932	Water consumption	Yes	Yes
234	Gilmer	3,000	...	....	Water consumption	Yes	No
235	Greenville	14,000	...	....	Type of occupancy	Yes	Yes
236	Haskell	3,050	450	Feb. 17, 1927	Flat rate	Yes	Yes
237	Kerrville	6,000	1,050	Dec. 6, 1936	Type of occupancy	Yes	No
238	Fort Stockton	...	250	1934	Type of occupancy	Yes	Yes
239	Nacogdoches	8,000	...	1917	Type and number of fixtures	Yes	Yes
240	Paducah	2,802	495	June 10, 1937	Type of occupancy	Yes	Yes
241	Spur	2,100	...	1926	Number of connections	Yes	Partly
242	Tyler	28,256	...	1934	Type and number of fixtures	Yes	Yes
243	Vernon	10,000	...	....	Flat rate	Yes	Yes

221. *Arlington.*—Charges, 25¢ per month per connection; monthly bills.
222. *Ballinger.*—Sewerage system is owned privately; minimum charges are $1.25 for three fixtures; minimum charge, $3.25 per quarter; quarterly bills.
223. *Brady.*—Graduated charge, beginning at 25¢ per month; monthly bills.
224. *Canadian.*—Charge per residence, 50¢ per month; for business, $1.25 per month; monthly bills.
225. *Celeste.*—Original ordinance was amended June 3, 1940; charges, $1.00 per month minimum; monthly bills.
226. *Childress.*—Charges based on a graduated schedule beginning at 50¢ per month; monthly bills.
227. *Commerce.*—Charges, 90¢ flat rate per connection; monthly bills.
229. *Crowell.*—Flat rate of 50¢ per month for dwellings and $1.00 per month for business; monthly bills.
230. *Dimmitt.*—Charge based on water consumption and type of occupancy; minimum residence rate, $2.00 monthly; monthly bills.
232. *El Paso.*—Texas has by far the largest number of cities employing sewer rentals. Out of approximately 325 cities to whom questionnaires were sent, replies were received from only about 25. Charges are quite uniform throughout; monthly bills. El Paso may be used as a typical example as follows:

Minimum charge of 25¢ based on amount of water used according to the following schedule—

First 500 cu ft or less	$0.25
Next 500 cu ft	0.05 for each 100 cu ft
Next 2,000 cu ft	0.03 for each 100 cu ft
Next 5,000 cu ft	0.02 for each 100 cu ft
Next 42,000 cu ft	0.01
50,000 cu ft	0.00½
All over 100,000 cu ft	0.00¼

Where charge is made according to type and occupancy, Fort Stockton may be cited:

[d] "A and I" denote "amortization and interest, respectively, on sanitary sewers and sewage treatment plants."

TABLE 3.—(*Continued*)

No.	Municipality[a]	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]

(z) Texas (*continued*)

Basis of charge

Residence, first 4 or less fixtures.... $1.50 per month
Stores, etc., first 2 fixtures or less... 1.50
Manufacturing establishments...... 5.00 per month minimum
20% discount is allowed.

233. *Fort Worth.*—Minimum charge, 25¢ per month; monthly bills.
234. *Gilmer.*—Minimum charge, 25¢ per month; monthly bills.
235. *Greenville.*—Minimum charge for residences, 25¢ per month; for commercial establishments, 30¢ per month; monthly bills.
236. *Haskell.*—Minimum rate for residences, $1.25 per month; monthly bills.
237. *Kerrville.*—Minimum rate, 50¢ per month; monthly bills.
238. *Fort Stockton.*—Minimum rate for residences, $1.50 per month, less 20%; monthly bills.
239. *Nacogdoches.*—Minimum rate for residences, 75¢ per month; and for business, $1.00 per month; monthly bills.
240. *Paducah.*—Minimum rate for residences, 25¢ per month; and for business, $1.00 per month; monthly bills.
241. *Spur.*—Minimum rate, 75¢ per month per connection; monthly bills.
242. *Tyler.*—Minimum rate: 50¢ per month for the first fixture plus 10¢ per month for each additional fixture; monthly bills.
243. *Vernon.*—Minimum rate for residences, 50¢ per month; and for business, 75¢ per month; monthly bills.

(*aa*) Utah

244	Kaysville	1,600[b]	...	Nov. 5, 1934	Flat rate	Yes	Yes

244. *Kaysville.*—Population equivalent 1,100 + 1,600 due to high school. Minimum residence charge, $1.50 monthly, and minimum commercial charge, $2.00 monthly; monthly bill.

(*bb*) Washington

245	Elma	1,000	320	Nov. 4, 1935	Flat rate	Yes	Yes
246	Selah (T)	750	320	Nov. 12, 1935	Flat rate	Yes	Yes

245. *Elma.*—Residences are charged a flat rate of 50¢ per month, and businesses, $1.25 per month; monthly bills.
246. *Selah.*—Charges monthly, $1.25 per complete house.

(*cc*) Wisconsin

247	Oshkosh	40,108	...	....	Water consumption[b]	Yes	Yes
248	Stevens Point	14,000	...	....	Water consumption	...	...
249	Sturgeon Bay	4,500	...	1938	Size of water meter[b]	Yes	Yes

247. *Oshkosh.*—Charges are based principally on water consumption; that is, a graduated schedule with a minimum charge of 40¢ per quarter; quarterly bills.

TABLE 3.—(*Continued*)

No.	Municipality[a]	Population	Connections	Date of ordinance	Basis of charge[b]	Rental Used for: M and O[c]	Rental Used for: A and I[d]
	(cc) Wisconsin (*continued*)						

248. *Stevens Point.*—Rental rate, 70% of water charges; quarterly bills.
249. *Sturgeon Bay.*—Rental rate based on a service charge plus the size of the water meter. Minimum charge, 50¢ per month; monthly bills.

No.	Municipality	Population	Connections	Date of ordinance	Basis of charge	M and O	A and I
	(dd) Wyoming						
250	Lusk	1,000	...	Dec. 6, 1938	Classification	Yes	Yes

250. *Lusk.*—Minimum residential charge, 60¢ per month; monthly bills.

[a] "T" denotes "town," "C" denotes "city," "V" denotes "village," and "B" denotes "borough."

[b] Charges in given cases and an explanation of rates are given with each state section.

[c] "M and O" denote "maintenance and operation, respectively, of sanitary sewers and sewage treatment plants."

[d] "A and I" denote "amortization and interest, respectively, on sanitary sewers and sewage treatment plants."

Table 4 lists municipalities operating under sewer rentals from which the Committee has not received replies.

TABLE 4.—Municipalities Reported to Have Sewer Rentals In Effect But From Which No Replies Were Received

(*a*) Alabama

Alexander City, Allenville, Ashland, Carbon Hill, Childersburg, Cordova, Cullman, Gordo, Grove Hill, Guin, Irondale, Jackson, Jacksonville, Leeds, Linden, Livingston, Moundville, Sulligent, Springville

(*b*) Arkansas

Alma, Batesville, Bearden, Berryville, Brinkley, Carlisle, Charleston, Conway, Cotton Plant, Desarc, Douglas, Dumas, Eureka Springs, Fordyce, Gentry, Glenwood, Harrison, Heber Springs, Judsonia, Kensett, Little Rock, Parkin, Rison, Russellville, Searcy, Siloam Springs, Star City, Stephens, Stuttgart, Waldo, Waldron

(*c*) Florida

Clearwater, Fort Lauderdale, Hollywood, Leesburg, Milton, River Junction

(*d*) Illinois

Abingdon, Benton, Bradley, Du Quoin, Edwardsville, Effingham, Gillespie, Hillsboro, Ladd, McHenry, Nashville, Polo

(*e*) Indiana

Beech Grove, Bloomington, Evansville, Goshen, Greenfield, Huntington, Kokomo, Lawrenceburg, Lebanon, Michigan City, North Vernon, Princeton

(*f*) Kentucky

Elizabethtown, Middlesborough, Murray, Providence, Russellville, Scottsville

(*g*) Michigan

Alma, Ann Arbor, Battle Creek, Bessemer, Caro, Charlevoix, Chelsea, Detroit, Gladstone, Grand Haven, Grayling, Jackson, Lawton, Ludington, Manistee, Menominee, Monroe, Muskegon, Muskegon Heights, Paw Paw, Pontiac, Rochester, St. Ignace, Spring Lake, Traverse City, Ypsilanti

TABLE 4.—(*Continued*)

(*h*) SOUTH CAROLINA
Bamberg, Calhoun Falls, Fort Mill, Hartsville, Greenwood, Rock Hill, Sumter, Williamston
(*i*) TENNESSEE
Brownsville, Bruceton, Centerville, Cookeville, Decherd, Etowah, Lawrenceburg, Parsons, Selmer, Sevierville
(*j*) TEXAS
Arp, Big Sandy, Bridgeport, Cameron, Claude, Canton, Corpus Christi, Cushing, Daingerfield, Dayton, Decatur, Edgewood, Elkart, Fairfield, Follett, Fort Stockton, Fredericksburg, Frisco, Glen Rose, Goose Creek, Granbury, Grapeland, Groveton, Hughes Springs, Kosse, Lewisville, Lindale, Linden, Marshall, Meridian, Monahans, Mount Calm, Muleshoe, Newcastle, Pasadena, Pearsall, Pecos, Pilot Point, Port Lavaca, Stanton, Weslaco, Wheeler, Whitney, Woodsboro
(*k*) WEST VIRGINIA
Beckley, Elizabeth, Marmet, Mason, Matewan, Parkersburg, Peterstown, Point Pleasant, Romney, Vienna
(*l*) MISCELLANEOUS
Arizona: Mesa; Iowa: Iowa City, and Parkersburg; Louisiana: Bayville, and Sulphur; Mississippi: Duck Hill, and Nettleton; Nebraska: Trenton; New Mexico: Belen, Clovia, and Silver City; North Dakota: Cavalier; Oregon: Lakeview, and Nyssa; Utah: Farmington; Virginia: Berryville, Williamson Road, and Virginia Beach; Washington: Everson, and Poulsbo; and Wisconsin: Seymour.

APPENDIX 2

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DESIGN OF ST. GEORGES TIED ARCH SPAN

Discussion

BY HAROLD E. LANGLEY, ESQ.

HAROLD E. LANGLEY,[14] ESQ.[14a]—The tied arch design adopted for the St. Georges bridge is certainly an unusual type in the United States. There seems no doubt that, prior to World War II, there has been a definite trend in design toward the selection of structures which meet esthetic as well as utilitarian requirements. A pleasing design is usually the product of the successful solution of perplexing engineering problems. Professor Garrelts' paper, therefore, is an important contribution to the engineering profession.

Formulas based on the elastic theory seem to have been accurate enough for the design of this bridge, as the results agree with those secured by a model analysis. Inasmuch as both the girder and arch rib are relatively slender members, considerable deformation under load is likely to occur. Because of this tendency, the writer feels that formulas based on the deflection theory would prove more satisfactory, especially if the means of checking the results by a model analysis is not available. In the case of suspension bridges, a design based on the elastic theory requires more material than one based on the deflection theory. The deflection theory does not permit the use of influence lines and, therefore, is somewhat more laborious to apply. Because of the unusual proportions of the bridge under discussion, the writer feels that, if he were called upon to design a similar structure, he would apply the deflection theory.

Although no reference was made to secondary stresses in Professor Garrelts' paper, they were undoubtedly considered in the design. Relatively small bending moments in the arch ribs, in combination with the axial loads, may produce buckling stresses which approach serious proportions. The writer found this to be true in the design of a two-hinged arch, as well as a tied steel arch, each of which had a 425-ft span. As the tied arch had ribs and ties of more conventional proportions than those of the St. Georges bridge, the

NOTE.—This paper by J. M. Garrelts, Assoc. M. Am. Soc. C. E., was published in December, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1942, by R. W. Abbett, M. Am. Soc. C. E.; April, 1942, by Jacob Karol, Esq.; May, 1942, by Alexander Dodge, Esq.; June, 1942, by Carlos A. Bejarano, Jun. Am. Soc. C. E.; September, 1942, by Messrs. A. A. Eremin and A. M. Freudenthal; and October, 1942, by C. H. Gronquist, Assoc. M. Am. Soc. C. E.

[14] Engr. of Design, Highway Dept., State of New Hampshire, Concord, N. H.

[14a] Received by the Secretary September 25, 1942.

buckling effect in the latter design might be of major importance. This appears to be very probable, as Fig. 6 indicates that the girder ties and the arch ribs are rigidly connected. If this is so, considerable bending must be transmitted to the ribs from the ties, due, at least, to live load.

These comments are not intended in any way as a criticism of a very excellent paper, but should be regarded as a brief résumé of some design features that are likely to be slighted but which are of sufficient importance to merit a thorough investigation. Professor Garrelts is to be commended for bringing the attention of the profession to an unusual but pleasing type of bridge and an interesting solution of its design.

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DISCUSSIONS

DRAINAGE OF LEVEED AREAS IN MOUNTAINOUS VALLEYS

Discussion

BY GORDON R. WILLIAMS, ASSOC. M. AM. SOC. C. E.

GORDON R. WILLIAMS,[24] Assoc. M. Am. Soc. C. E.[24a]—The volume of discussion pertaining to this paper was gratifying to the writer, particularly since the subject was not one to arouse interest during wartime. Many interesting angles were stressed, and considerable space could be devoted to developing further the points in question. There was general agreement that the title of the paper was too restricted and that the principles presented have wide application to drainage problems, both urban and rural, in many parts of the United States.

In discussing the methods of disposing of local drainage, Mr. Bernard proposed an additional method, or rather a combination of methods, in which a secondary levee and pumping station would be provided upstream from the main river levee and pumping station. Such a method might be worthy of consideration under special conditions, but the use of two sets of pumping installations and gates rarely would be justified from an economic standpoint.

The importance of having adequate levee heights under method (1) was demonstrated by Mr. Sherman with an example. The writer has utilized the following method for developing adequate height for tributary levees:

(*a*) From known records of discharges, develop a relation between coincident flood discharges on the main river and on the tributary;

(*b*) Compute flow lines up the tributary, using a range of coincident discharges; and

(*c*) Draw an envelop of the flow lines.

The envelop curve plus a normal freeboard for wave action will constitute the levee profile. The height of the downstream end will be governed by the

NOTE.—This paper by Gordon R. Williams, Assoc. M. Am. Soc. C. E., was published in January, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1942, by Merrill Bernard, M. Am. Soc. C. E.; April, 1942, by L. K. Sherman, M. Am. Soc. C. E.; May, 1942, by Messrs. C. O. Clark, W. W. Horner, and Walter T. Wilson; and June, 1942, by W. B. Langbein, Assoc. M. Am. Soc. C. E.

[24] Area Engr., U. S. Engr. Dept., Wilkes-Barre, Pa.

[24a] Received by the Secretary October 13, 1942.

magnitude of the design discharge in the main river, but the height of the upstream end usually will be governed by the magnitude of the design discharge in the tributary. In this connection, Mr. Sherman took exception to the statement that "A corresponding degree of protection is not required against flows from tributaries that pass through leveed areas." This statement was intended to apply particularly to the capacity of structures such as pressure conduits and pumping stations. The writer agrees with Mr. Sherman that in the application of method (1) the statement should not be taken too literally and that adequate upstream height is well worth the relatively small additional cost. On other types of drainage structures, it is obviously not sound engineering practice to provide discharge capacity for a flood having a probable frequency of occurrence of once in a hundred years or less.

Mr. Sherman commented on the high pumping capacities in inches per day, shown in Fig. 5, as compared with pumping capacities found necessary for leveed areas on the Illinois and upper Mississippi rivers. The latter areas are flat, provide much more storage per square mile of tributary area, and have a resulting delayed runoff. The pumping stations along the Susquehanna River must dispose of a hydrograph resulting from quick runoff from mountain slopes. Records of flood runoff coincident with high stages in the Susquehanna River already have been obtained on one area of 18 sq miles and indicate that a pumping capacity of 0.8 in. per day in connection with more than 2.0 in. of flood-plain storage will be required to meet conditions which have already been experienced and which were not unusual. Such a capacity is in close agreement with the mean curve in Fig. 5.

Several of the discussers questioned the assumed rate of snow melt of 1 in. in 24 hr in connection with the design storm. Messrs. Bernard and Wilson considered the rate too low, whereas Mr. Horner considered that the use of 100% runoff plus snow melt is too conservative. The writer admits that there were few data upon which to base the assumption, but is interested to note the wide divergence of opinion among recognized specialists in hydrology. The selected rate of snow melt was intended to have the same probable frequency of occurrence as the design storm rainfalls—namely, 5 to 15 years. Actually, the rate of snow melt, which is only 0.04 in. per hr, has little effect on the peaks of the hydrographs and is merely inserted for its influence on the 1-day and 2-day storage requirements.

Mr. Wilson took exception to the statement that "Flood protection from small tributaries is very rarely justified," and stated that the writer "has exposed himself to criticism by proponents of upstream engineering and flood control in source regions." In making the statement the writer has reference to the economic justification for the erection of expensive structures, such as large storage dams, when the benefits are to accrue only to the tributaries themselves. The burden of proof of the efficacy as well as the justification of other methods, such as terraces, check dams, and crop control, is squarely up to the proponents of such methods. The determination of the annual charges for such work will not be difficult, but the computation of the reduction in flood

stages at points downstream for floods of different magnitudes and frequencies will test the ingenuity of agricultural engineers and agronomists.

The use of the rational formula (Eq. 2) to determine the peak of the time of concentration for the unit hydrograph received considerable comment. Mr. Langbein and Mr. Sherman were of the opinion that the writer was attempting to establish a new definition for time of concentration, but that was not the case. The confusion seems to result from the fact that the writer was concerned only with rainfall that would run off—that is, rainfall excess. In accordance with the rational theory, if a uniform rainfall falls on saturated soil and continues to fall until all parts of the area are contributing, the theoretical rate of runoff will equal the assumed uniform rainfall rate and will continue at that rate as long as the rainfall continues. Mr. Horner pointed out that the C in the rational formula, as generally used, involves both losses and delay factors. By using a time of concentration derived from actual stream-flow records, the writer has introduced, in part at least, some of the delay factors. Some of the conservatism derived from the use of the rational theory is nullified in the application of the final flood hydrographs, which are greatly modified by routing through existing storage and constrictions. It is agreed that the method of deriving the unit hydrograph is conservative, and should not be resorted to if actual records are available.

Messrs. Horner and Langbein presented methods of deriving synthetic unit hydrographs by correlation of lag time with the characteristics of recorded hydrographs. The writer believes that the principles involved are perhaps superior to those presented in his paper, but are difficult to apply for the following reasons:

(*a*) Records of complete hydrographs from areas of less than 10 sq miles are difficult, if not impossible, to obtain for a wide variety of drainage area characteristics; and

(*b*) The extrapolation and application to very small areas of relations obtained from hydrographs for relatively large areas will lead to erroneous results and to serious under-design.

Mr. Clark considered, in detail, studies for determining rainfalls expected to occur coincident with different river stages. The writer agrees that this is the ideal solution for drainage areas tributary to large rivers or at any locality where there are sufficient records. The results of a study of this kind have been presented by Frank A. Marston,[25] M. Am. Soc. C. E. Rainfalls coincident with different river stages were considered in Mr. Horner's discussion, where he outlined the problem of the design of a pressure culvert at Dallas, Tex.

The expression "maximum probable" flood is questioned by Mr. Wilson, and he prefers the term "maximum possible" flood. The writer agrees that his expression involves two words which are somewhat contradictory, but what he attempted to determine was an extraordinary flood which by nature could not be defined within close limits. If the "maximum possible" flood could be determined, it would have no application to the structures under consideration,

[25] *Civil Engineering*, July, 1939, pp. 409–411.

and would have little application to engineering design except in the determination of reserve spillway capacity for very large dams. The advisability of using even the "maximum probable" flood in determining the capacity of relief openings is questionable, except where great damage will result from backwater. As stated in the description of method (2), it will be found in many cases to be good engineering and sound economic practice to plan to blow a hole in earth levees in the case of a very rare occurrence rather than spend large sums on oversized gates and culverts.

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DISCUSSIONS

ANALYTICAL AND EXPERIMENTAL METHODS IN ENGINEERING SEISMOLOGY

Discussion

BY M. A. BIOT, ESQ.

M. A. BIOT,[53] ESQ.[53a]—Mr. Rich presents an application of the operational calculus to a simplified case. Since the response of structures to earthquakes is essentially of a transient character, it is natural to apply the methods specifically designed for the treatment of such phenomena. However, the difficulty in applying the Heaviside methods to the full extent is the non-analytical and random character of the seismogram. The simplified procedure derived by the writer avoids the computation of the actual motion and makes possible the direct evaluation of the maximum stress by the use of a standardized spectrum. However, the operational method can be used, as such, in combination with the spectrum, as a convenient way of deriving the effectiveness coefficients or any desired characteristic in the structure. This is especially true when the information needed is restricted to a particular variable or location in the structure. The operational method is also useful in investigating the response to simplified theoretical earthquakes, as shown in the examples set forth by Mr. Rich. The latter method acquires special value if a great number of cases are solved and if general principles are uncovered as to the comparative behavior of structures under standardized earthquakes. This approach might truly be called a mathematical, experimental method.

Professor Hoff mentions that the results obtained at Stanford University show the influence of the friction on the oscillator response. It is interesting to note the experimental fact that the response is very sensitive to changes in friction when the latter is small. It is probable that the differences in peak spectrum accelerations between the Stanford results and those of the writer are partly due to errors on the small-period components of the displace-

NOTE.—This paper by M. A. Biot, Esq., was published in January, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1942, by Messrs. George R. Rich, N. J. Hoff, and Merit P. White; May, 1942, by Messrs. N. H. Heck, Frank Neumann, and Jacob Feld; June, 1942, by Messrs. George W. Housner, and Homer M. Hadley; and September, 1942, by Robert E. Glover, Esq.

[53] Research Assoc. in Aeronautics, California Inst. of Technology, Pasadena, Calif.; Asst. Prof. of Mechanics, Physics Dept., Columbia Univ. (on leave of absence).

[53a] Received by the Secretary October 22, 1942.

ment cam. The question of the possible accumulation of the stresses in each mode by the simultaneous occurrence of their maximum values at a given point is well worth investigating. A conservative estimate can be made theoretically by adding the maximum values associated with each mode at various locations in the structure.

As stressed by Professor White, the idea of an automatic analyzer to avoid the numerical work in evaluating the response of a structure is not new. In the writer's earlier work in 1932, the mathematical groundwork was laid for showing the possibility of using the accelerogram directly without the process of evaluating the displacement. An electric model was suggested, of which the present torsional pendulum is the mechanical analogue. An electric model has been developed by Arthur C. Ruge, Assoc. M. Am. Soc. C. E., at the Massachusetts Institute of Technology in Cambridge, Mass.

Professor White raises an interesting and important point regarding the nature of the peaks in the spectrum. The writer called attention to the lack of correlation between the peaks in the two Ferndale earthquakes, (*B*) and (*C*), as supporting evidence that these peaks are not characteristic of the location. It is possible that some peaks might be characteristic of the location, or of a group of buildings, and that these would stand out particularly in earthquakes of small intensity where the effects of resonance seem to be more acute. More probably the hatched appearance of the spectrum is due to random transients superposed on a basic periodicity distribution. Professor White makes the statement that the whip effect, although less pronounced, also appears in structures that are not necessarily tapered. He also suggests the application of the present results to the effect of explosions and projectile impact on structures. The writer wishes to mention another possible application—determining the dynamic effect of wind on structures by introducing the spectrum of atmospheric turbulence.

Mr. Heck mentions the importance attached by the U. S. Coast and Geodetic Survey to the recording of high accelerations throughout the frequency range of earthquakes. The results already accomplished in this direction were an absolute prerequisite to the writer's work.

Mr. Neumann rightly warns against using a particular spectrum as a basis for final standardization. The writer uses the standardized Helena spectrum merely as an example. The spectrum peak seems to be correlated both in intensity and frequency to the distance of the epicenter. As stated by Mr. Neumann, it is probably true that the torsion pendulum analyzer is not necessarily less accurate than a more intricate electrical apparatus. The writer's statement on this matter was perhaps somewhat conservative, since the spectrum curves could be duplicated closely in spite of the rather crude design of the analyzer and the use of non-magnified seismographic records.

Mr. Feld recalls the change of damping characteristics with time in a vibrating soil. As stated by the writer, the treatment of a foundation is restricted to its elastic properties. The results are in good agreement with

tests on actual foundations made by Mr. Barkan.[16] The magnitude of structure and soil damping is a factor left to future experimental investigation. The methods of the paper are applicable if it is desired to introduce the effect of this damping on the amplitudes of oscillation. However, in seeking to establish experimental data, it will be well to keep in mind that the vibrator itself can have a considerable effect on the properties of the soil.

The two spectra computed by Mr. Housner are interesting, in that they show the same general appearance as those obtained by the writer. The ratio of the spectrum peak to the maximum acceleration is also of the same order. The spectrum of the March 10, 1933, earthquake shows relatively high equivalent accelerations (of the order of 20% gravity) for the large periods. Mr. Housner presents an application of the writer's simplified formula for the rocking period of blocks. His results are based on a much lower bearing pressure than that used by the writer, and the rocking periods in this case are relatively short. Taking into account the possibility of the soil being considerably more spongy than assumed in the computations, it should be concluded that the effect of the foundation is important or not, depending on the magnitude of such factors as soil elasticity, foundation size, and height and rigidity of the building. It would seem, however, that in the case of towers or chimneys the rocking effect should be preponderant. In regard to the relations between acoustics and engineering seismology, a theorem[4,54] was originally developed by the writer which states that the energy accumulated by an oscillator depends only on the intensity of the impulse spectrum for the particular frequency. This theorem forms the basis of the present method and justifies the use of the spectrum.

Mr. Hadley's pimented discussion is enjoyable reading. His argument reduces to a confession of man's total ignorance of the subject, and the defeatist viewpoint that nothing can be done about it "Now, and probably for a long time in the future." Mr. Hadley, however, is anxious not to discredit the esoteric art of earthquake-proof design by contributing the following statement:

> "* * * after the earthquake has passed, a well-designed, rigid-type building will be found very satisfactory to all concerned. * * * As to skyscrapers, the writer has no knowledge other than that such buildings successfully passed through the San Francisco, Calif., earthquake of 1906."

This, said the medieval doctor, is a sleeping potion; it makes you sleep because it contains a soporiferous virtue. It has been the writer's experience that a feeling of uneasiness usually develops when proselytes of the empirical method are brought in contact with symbols such as E and μ, even if they do not represent an essential part of the argument. This is entirely unfortunate and unjustified, as there is no fundamental conflict between empiricism and science.

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[16] "Field Investigations of the Theory of Vibrations of Massive Foundations Under Machines," by D. D. Barkan, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Vol. II, 1936, p. 285.

[4] "Theory of Elastic Systems Vibrating Under Transient Impulse with an Application to Earthquake Proof Buildings," by M. A. Biot, *Proceedings*, National Academy of Science, Vol. 19 (1933), pp. 262-268.

[54] "Acoustic Spectrum of an Elastic Body Submitted to a Shock," by M. A. Biot, *Journal*, Acoustical Soc. of America, Vol. V, January, 1934, p. 207.

Mr. Hadley's objections might well have been raised three hundred years ago against the basic principles of Newtonian mechanics with their abstract and elusive concepts of force and mass.

Mr. Glover's results are in good agreement with those of the writer. They point to the comforting fact that satisfactory duplication of results is possible even when different methods and instruments are used.

In conclusion, the writer wishes to thank all those who have taken time to contribute to the present discussion for their constructive criticism.

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DISCUSSIONS

THE GREASE PROBLEM IN SEWAGE TREATMENT

Discussion

By F. M. Dawson, M. Am. Soc. C. E.

F. M. Dawson,[16] M. Am. Soc. C. E.[16a]—In discussing the various methods of grease removal from sewage, the authors made brief mention of removing grease at the source of the sewage by use of interceptors. The writer believes that this method of grease removal merits greater emphasis and consideration than hitherto it has received. This is especially true at present, when it is so vital to save all greases and oils. Naturally, reclaimed grease and oil will be in the best condition for further use if they are removed from sewage as soon as possible.

The common type of grease interceptor has not proved entirely satisfactory in the past, owing to the fact that proper consideration was not given to its design, installation, and cleaning. Hitherto a fixture was installed frequently because some code or regulation required it, and then it was more or less forgotten. However, the basic principle of separating the grease from the water as close to the source as possible is sound. All that was lacking was good engineering in the design and installation of interceptors, and proper maintenance once they were installed.

Contrary to popular belief, the grease is not separated from the incoming waste water, as a result of a reduction in temperature as the water flows through. The grease must be separated in a liquid state since sufficient time is not available for cooling, and also since the temperature of the interceptor is almost always, when in operation, above the congealing temperature of grease. The separation is obtained by securing a proper reduction in velocity of the entering water by use of baffling and by preventing the occurrence of large-scale turbulence. Proper separation depends mainly upon the density difference between the grease and the water. Well-designed interceptors will maintain an average efficiency of 90% for a flow rate equal to 1 gal per min for each 1.0 to 1.5 gal of interceptor capacity. The proper design of interceptors, however,

Note.—This paper by Almon L. Fales and Samuel A. Greeley, Members, Am. Soc. C. E., was published in February, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1942, by Arthur D. Weston, M. Am. Soc. C. E.; May, 1942, by Harvey F. Ludwig, Jun. Am. Soc. C. E.; June, 1942, by W. S. Mahlie, Esq.; and September, 1942, by Messrs. Denis Dickinson, and Charles R. Velzy.

[16] Dean and Prof. of Eng., Univ. of Iowa, Iowa City, Iowa.

[16a] Received by the Secretary September 23, 1942.

involves the application of considerable experience with, and knowledge of, fluid flow. It is well known that the general problem of separating fluids of different densities is one of importance in many manufacturing plants. The particular case of grease and water is in principle little different from many others, but it is greatly complicated by the many types of wastes carried from the kitchen sink or other sources of waste water.

The government is installing commercial-type interceptors in the kitchens of all cantonments and many other army and navy establishments. To insure that the interceptors will perform properly, each type should have a laboratory test certificate. The Iowa Institute of Hydraulic Research at the University of Iowa, in Iowa City, in cooperation with the Office of the Chief of Engineers and manufacturers of grease interceptors, has developed a laboratory-test procedure and a rating method which indicate quite well the field performance of interceptors. By this procedure interceptors are rated according to the rate of flow, in gallons per minute, which they can handle and still retain, on an average, 90% of the incoming grease. From tests on a great many interceptors, it was decided also that, for the stated flow rate, an interceptor should maintain an average efficiency of 90% up to a grease-retaining capacity equal, in pounds, to twice the flow rate in gallons per minute. Thus, a 25-gal-per-min interceptor should maintain an average efficiency of 90% with a grease capacity of 50 lb.

It is hoped that the war needs for grease and oil will emphasize the many advantages of separating grease and oil at the source, and thus interest more hydraulic and sanitary engineers in the proper design and installation of grease and oil interceptors. All restaurants, hospitals, hotels, and similar types of commercial kitchens should have interceptors near their sinks. For the ordinary restaurant kitchen sink, the flow-rate capacity of any interceptor should not be below 15 gal per min, and preferably not below 20 gal per min. Regular cleaning, at least once a week, is of prime importance.

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DISCUSSIONS

STRESS CONCENTRATIONS IN PLATES LOADED OVER SMALL AREAS

Discussion

BY MESSRS. D. L. HOLL, AND C. A. P. TURNER

D. L. HOLL,[42] ESQ.[42a]—Any attempt to analyze the stresses that arise in areas of a plate which are subjected to high concentration of normal loading is subject to the limitations of the theory used. The author carefully has presented the hypothesis of the Poisson-Kirchhoff theory, which is used in the derivation of the Lagrange plate equation, a fourth-order partial differential equation for the deflection $w(x,y)$. This equation is consistent with the requirement that the total strain energy should arise only from flexural and torsional stresses which are supposed to be linearly distributed through the thickness h of the plate. All other stress considerations are excluded.

In the plate equation,

$$\nabla^4 w = \nabla^2 \nabla^2 w = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$
$$= \frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2\, y^2} + \frac{\partial^4 w}{\partial y^4} = f(x,y) \qquad (157)$$

w denotes the deflection, x and y are the rectangular coordinates of a point, and $f(x,y)$ denotes the loading function (this includes the stiffness factor N). If the plate bears only a concentrated load P, then the right member of Eq. 157 is zero everywhere except at the load point, where it has a singularity which is characterized by the integral,

$$\int - N\,\frac{\partial}{\partial n}(\nabla^2 w)\, ds = -P \qquad (158)$$

in which $\frac{\partial}{\partial n}$ denotes the outward normal derivative evaluated along any path which encloses the loading point, and ds is an element of arc on this path. The

NOTE.—This paper by H. M. Westergaard, M. Am. Soc. C. E., was published in April, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1942, by Messrs. George R. Rich, Dana Young, and A. Nádai; and October, 1942, by Messrs. L. A. MacColl, and L. W Teller and Earl C. Sutherland.

[42] Prof., Dept. of Mathematics, Iowa State College, Ames, Iowa.

[42a] Received by the Secretary August 31, 1942.

physical meaning of this integral is easily understood, since it indicates that the total vertical shearing forces on any closed path surrounding the load is statically equivalent to the load P. If it is found that the solution of a differential equation contains a singularity of such a nature that one or more of the derivatives of the function fail to exist, or even the function itself, then the corresponding expressions are without meaning and a precise formulation of the result is not possible. This is not a weakness of the theory but a consequence of the mathematical abstraction—namely, the representation of a point load by some type of function which shows a sudden discontinuity.

Not infrequently a point load is considered as the limiting case of allowing a uniform pressure distribution over a finite area to increase indefinitely in magnitude as the area becomes infinitesimal in such a manner that the product of pressure times area remains finite and equal to the load P. As an example, a pressure p_o is uniform over a circle of radius c; then a concentrated load is

$$P = \underset{\left[\begin{array}{l} p_o \rightarrow \infty \\ c \rightarrow 0 \end{array}\right]}{\text{limit}} \pi\, p_o\, c^2 \ldots\ldots\ldots\ldots\ldots\ldots (159)$$

The theory of elasticity is based upon the hypothesis that the induced stresses and strains are limited to those which are admissible when the linear relationship is given by Hooke's law. If the area over which a load is distributed is decreased indefinitely, and at the same time the resultant load is to remain constant, then the basic assumption of this law is violated. This does not imply that such concepts as concentrated forces or moments find no place in the theory, but rather that by introducing the fiction of a point load one can obtain more general results at distances somewhat removed from the zone of high stress concentration. It is known that the latter values are valid by Saint Venant's principle. Another important use of the concept of a concentrated load is made when one knows that the principle of reciprocity holds for both flexural and twisting moments as well as for deflections.

The specific contribution of this paper is the fact that the stresses arising from the bending and twisting moments which are induced in a thin plate by normal loads can be characterized to a fair degree of accuracy by two sets of dimensionless numbers called "place coefficients" and "area coefficients." The latter are independent of the former and depend only on the size and shape of the loaded area and on the distribution of load over that area. These area coefficients are defined by

$$K = \frac{-1}{P} \int_A p \log \frac{R}{c}\, dA \ldots\ldots\ldots\ldots\ldots\ldots (160)$$

and

$$S, T = -\frac{1}{P} \int_A p\, (\cos 2\,\alpha, \sin 2\,\alpha)\, dA \ldots\ldots\ldots\ldots (161)$$

in which dA is an element of the loaded area A over which the total load P has a local pressure p, and R is the distance from the element at (u,v) to the point x,y where the stress intensity is desired. The angle α is given by

$$R \sin \alpha = y - v, \qquad R \cos \alpha = x - u \ldots\ldots\ldots\ldots (162)$$

The area coefficient K in Eq. 160 may be defined in terms of a potential problem. It gives the logarithmic potential at (x,y) of an infinitely long rod of section A over which the mass distribution is of density p. One should note that the quantity c in Eq. 160 can be chosen arbitrarily and is generally some representative length either of the plate or of the area of loading, and thus the value of K is not an absolute number, but a relative one. This coefficient also satisfies the differential equations

$$\nabla^2 K = 0 \text{ (in regions of no loading)} \qquad (163a)$$

and

$$\nabla^2 K = -\frac{2\pi p}{P} \text{ (within regions of loading)} \qquad (163b)$$

In general, the determination of the value of K at an arbitrary point is not an elementary problem. Its existence is conditioned by the nature of the loading, but more specifically by the corners and sharp angular points along the boundary, and by line distributions. That these difficulties are recognized by the author is shown by his listing of three conditions for the radially symmetric case (paragraph following Eq. 102), three conditions for the general case (Eq. 106), and three additional observations (Eqs. 107 to 116) when analytic functions of complex variables are used. It is the writer's opinion that a greater use of the mathematical tool of analytic function theory does not make the problem more difficult, but rather tends to clarify it.

The author uses three "place coefficients," B, C, and D, which depend upon the size and shape of the plate, upon the conditions of support at the edge, and upon the position of the load relative to the edges; but these coefficients do not depend upon the load distribution. The reason for limiting them to three such coefficients is that it is assumed that one is interested in stress concentrations near small loading areas in which the deflection function can be suitably represented by a term containing an appropriate singularity and by polynomial terms of not more than the second degree. These place coefficients must be determined by examining known solutions of plate problems under given edge conditions. Since these place coefficients arise as coefficients of polynomial solutions of $\nabla^4 w = 0$ (that is, of biharmonic polynomials which are symmetric in the coordinate pairs (x,y) and (u,v)), the number of coefficients will usually be greater than three. The number depends upon the edge conditions, the shape of the plate, and the position of the load. If terms of third order are required to express the deflection of the plate, the author gives Eqs. 29 and 30 for the supplementary moments. These equations show the need of ten additional constants. In the six examples cited by the author, symmetry conditions yield a zero value for the place coefficient D and, of the remaining two, B is usually the more important one.

As stated at the beginning of this discussion, the stress concentration under a point load is infinite. To avoid this difficulty Dean Westergaard invokes the theory of elasticity in three dimensions, thereby considering all the stresses in a plate. Hence, when the loaded area is very small, "substitute coefficients," K', S', and T', for K, S, and T, respectively, are determined in such a way that the new coefficients define the correct stresses on the bottom surface of the

plate and directly under the center of the small area of loading. This amounts to the determination of an equivalent radius of loading such that, by using this new radius in the definitions of K, S, and T, the calculated stresses agree with the actual stresses as evaluated from the three-dimensional or thick-plate theory. It is the writer's opinion that Eqs. 129 and 130 are valid when there is no support on the lower surface of the plate near to the area of loading. One raises the question as to the validity of using this equivalent radius in a plate on an elastic subgrade. It would seem rational that such an equivalent radius should be a function not only of the actual loading radius and of the plate thickness, but also of the stiffness of the subgrade or of the ratio of relative stiffness of the plate to the subgrade. Until such an equivalent or substitute radius is found, one may accept the author's substitute coefficient K' for K, although he states that the corrected values S and T may be ignored in most practical problems.

The author also gives three examples in which the combination of area and place coefficient $(B + K)$ is corrected to $(B + K)'$ when it seems that the area of loading is too large to be rated as small. This determination of a new value for $(B + K)$ becomes rather difficult at any arbitrary point, but can be obtained at the center of the loading area if it is known that the reciprocal theorem for moments is valid. In view of the conditions limiting the use of these coefficients, it appears that the designing engineer will need to have a reasonably thorough understanding of their properties before he can utilize them in a new situation.

The writer wishes to supplement this discussion with some additional examples. The first two examples are to illustrate cases where the deflection of the plate contains polynomial terms far beyond the second degree.

Example 1. Clamped Square Plate with Central Point Load.—Let the side of the square be of length a, and the origin of the coordinate axes be at the geometrical center with axes directed parallel to the edges of the plate. The deflection is given by

$$w(x,y) = \frac{P}{8\,\pi\,N}\left[R^2 \log\frac{R}{a} + 0.5625 - 0.5562\,R^2 - \frac{0.0757}{a^2}(x^4 - 6\,x^2\,y^2 + y^4) + \frac{0.0654}{a^4}(x^6 - 5\,x^4\,y^2 - 5\,x^2\,y^4 + y^6) - \frac{0.00035}{a^6}(x^8 - 28\,x^6\,y^2 + 70\,x^4\,y^4 - 28\,x^2\,y^6 + y^8) + \cdots\right] \quad (164)$$

The place coefficients are $B = -0.4438$ and $C = D = 0$. All the polynomial terms of higher degree are solutions of $\nabla^4 w = 0$. One may readily compute the correction to the moments arising out of the supplemental terms.

Example 2. Centrally Loaded Square Plate Having Two Opposite Edges Clamped and the Other Edges Free.—The deflection is:

$$w(x,y) = \frac{P}{8\,\pi\,N}\left[R^2 \log\frac{R}{a} + 0.7783 + 0.0436\,R^2 - 0.3843\,(x^2 - y^2) + \frac{0.0183}{a^2}(x^4 - 6\,x^2\,y^2 + y^4) + \frac{0.2211}{a^2}(x^4 - y^4) + \cdots\right] \quad (165)$$

The place coefficients are $B = -1.0436$, $C = 0.7886$, and $D = 0$. Comparing these coefficients with a clamped infinite strip (Example *e* of the paper, in which $B = -1.044$, $C = 0.538$, and $D = 0$), one notes an apparent correlation in one case and a considerable divergence in the other. These coefficients also may be compared with the values $B = -0.4516$, $C = 1$, and $D = 0$ in Example *d* for a simply supported infinite plate strip symmetrically loaded by a single concentrated load. It is to be noted that, in the term $\log \frac{R}{a}$, the distance a can be chosen arbitrarily, but the choice of this distance will affect the value of B; the coefficient B attains its definite value after a has been chosen. In the foregoing examples, a is the width of the plate strip.

Example 3. Simply Supported Equiangular Plate; Concentrated Load at the Centroid.—Let the origin of a rectangular coordinate system be at the middle of an edge of the plate and the x-axis directed along an altitude of the triangular plate. Let the load be applied at $\left(\frac{a}{3}, 0\right)$, in which a is the altitude. By a summation process for the cumulative effects of an infinite number of alternating image loads, S. Woinowsky-Krieger[43] finds that the moments at points on the x-axis may be expressed with sufficient accuracy by

$$M_x, M_y = \frac{(1+\nu)P}{4\pi}\left[\log\frac{\sqrt{3}}{\pi} - 0.379 - \log\frac{R}{a}\right] \mp \frac{(1-\nu)P}{8\pi} \quad \ldots (166)$$

This yields a value of $B = -1.277$ and $C = D = 0$. If, in the term $\log \frac{R}{a}$, one uses the radius $c = \frac{a}{3}$ of the inscribed circle, then the expression in brackets in Eq. 166 is $\left(-0.179 - \log\frac{R}{c}\right)$ and B is -0.179. This indicates the change in the place coefficient B when the length c in the term $\log \frac{R}{c}$ is varied.

One may compare the moments in Eq. 166 with the moments M_r and M_t in a simply supported circular plate of radius a_o. These moments are:

$$M_r = -\frac{(1+\nu)P}{4\pi}\log\frac{R}{a_o} \quad \ldots (167a)$$

and

$$M_t = -\frac{(1+\nu)P}{4\pi}\log\frac{R}{a_o} + \frac{(1-\nu)P}{4\pi} \quad \ldots (167b)$$

Hence, the stresses in the equiangular plate near the load are smaller by $\frac{(1-\nu)P}{8\pi}$ than those in a circular plate of radius,

$$a_o = \frac{a\sqrt{3}}{\pi}e^{-0.379} = 0.279\,a \quad \ldots (168)$$

[43] "Berechnung der ringsum frei aufliegenden gleichseitigen Dreiecksplatte," by S. Woinowsky-Krieger, *Ing-Archiv*, Vol. IV, pp. 254–262.

If the load P is uniformly distributed over a circle of radius c centered at the centroid of the equiangular plate, then it is known[44] that at the center of loading the moments $M_x = M_y$ are greater by $\frac{P}{4\,\pi}$ than M_r at the boundary of the circle c. This is true for any symmetrical figure. Hence, in the triangular plate these maximum moments are:

$$M_x = M_y = \frac{(1+\nu)\,P}{4\,\pi}\left(\log\frac{a\sqrt{3}}{\pi\,c} + 0.121\right) = \frac{1+\nu}{4\,\pi}\left(\log\frac{a}{c} - 0.474\right) \quad (169)$$

S. Timoshenko[45] utilizes this superposition principle in finding the maximum moments under a circular loading centered upon a rectangular plate which has all its edges simply supported. These moments are:

$$M_x = \frac{P}{4\,\pi}\left[(1+\nu)\log\frac{a}{2\,c} + (1+\gamma_1)\right] \quad (170a)$$

and

$$M_y = \frac{P}{4\,\pi}\left[(1+\nu)\log\frac{a}{2\,c} + (\nu+\gamma_2)\right] \quad (170b)$$

in which a is the width of the plate, c is the radius of loading, and γ_1 and γ_2 are coefficients which depend upon the plate ratio $\frac{b}{a}$. For varying ratios, $\frac{b}{a}$, these coefficients are listed in Table 4.

TABLE 4.—VALUES OF PLACE COEFFICIENTS AND COEFFICIENTS γ, IN EQS. 170

Coefficient	PLATE RATIOS, $\frac{b}{a}$						
	1.0	1.2	1.4	1.6	1.8	2.0	∞
γ_1	−0.250	−0.035	0.103	0.190	0.242	0.273	0.314
γ_2	0.450	0.431	0.400	0.369	0.348	0.337	0.314
B	−0.616	−0.541	−0.500	−0.478	−0.466	−0.459	−0.452
C	0	0.34	0.58	0.74	0.85	0.91	1

The area coefficients K, S, and T, evaluated at the center of an area loading of radius c, are $K = \frac{1}{2} + \log\frac{a}{c}$ and $S = T = 0$. Comparing Eq. 170b with Eq. 13:

$$M_x, M_y = \frac{P}{4\,\pi}(1+\nu)(B+K) \pm \frac{P}{8\,\pi}(1-\nu)(C+S) \quad (171)$$

The B and C coefficients are listed in Table 4.

In the determination of these values of the place coefficients B and C, the writer has assumed that a value $\nu = 0.3$ was used by Professor Timoshenko in

[44] "Die elastischen Platten," by A. Nádai (Julius Springer, Berlin), 1925, p. 62.

[45] "Über die Biegung der allseitig unterstützten Rechteckigen Platte unter Wirkung einer Einzellast," by S. Timoshenko, *Bauingenieur*, Vol. 3, 1922, pp. 51–54.

obtaining the γ_1 and γ_2 values. This calculation yields the correct values of B and C for the infinite strip $\frac{b}{a} = \infty$ and for C when $\frac{b}{a} = 1$. The values of B and C in Table 4 are particular values of these constants for a centrally placed load, for rectangular plates with "pimeed" edges, and for a length c which is chosen as the radius of the circle over which the load is distributed. Furthermore one should use these coefficients only in Eq. 171 when determining the moments at the center of the loading area.

Application of Analytic Function Theory.—Let $\phi(z)$ and $\psi(z)$ denote analytic functions of the complex variable $z = x + i\,y$. Then $\bar{z} = x - i\,y$ is the conjugate variable and $\overline{\phi(z)}$ and $\overline{\psi(z)}$ denote the conjugate functions of $\bar{z}$ obtained from ϕ and ψ, respectively, by replacing all terms containing i by $-i$. The following results are readily obtained:

$$x = \tfrac{1}{2}(z + \bar{z}), \quad y = \frac{z - \bar{z}}{2\,i} \qquad (172a)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}, \quad \frac{\partial}{\partial y} = i\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}}\right) \qquad (172b)$$

and

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right), \quad \frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) \qquad (172c)$$

Since ϕ and ψ are analytic, both their real and imaginary parts satisfy $\nabla^2 = 0$, and a solution of a thin-plate problem having a single concentrated load can be put in the form:

$$w = \frac{P}{16\,\pi\,N}\left[z\,\bar{z}\log z\,\bar{z} + \phi(z) + \bar{z}\,\psi(z) + z\,\overline{\psi(z)} + \overline{\phi(z)}\right] \qquad (173)$$

Eq. 173 satisfies Eq. 157 when the right member of the latter is zero (that is, when $\nabla^4 w = 0$). The first term of Eq. 173 is the necessary singularity which satisfies Eq. 158. The entire expression for w in Eq. 173 is real. The functions ϕ and ψ can contain no singularities and one seeks to determine them by satisfying two boundary conditions along each arc of the boundary of the plate.

To meet various boundary conditions, the following reference formulas will be needed:

$$2\frac{\partial w}{\partial \bar{z}} = \frac{\partial w}{\partial x} + i\frac{\partial w}{\partial y} = \frac{P}{8\,\pi\,N}\left[z\,(1 + \log z\,\bar{z}) + \psi(z) + z\,\overline{\psi'(z)} + \overline{\phi'(z)}\right] \qquad (174)$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial z^2} + 2\frac{\partial^2 w}{\partial z\,\partial \bar{z}} + \frac{\partial^2 w}{\partial \bar{z}^2} = \frac{P}{16\,\pi\,N}\left\{\frac{z^2 + z^2}{z\,\bar{z}} + 4 + 2\log z\,\bar{z} + 2\,\mathrm{Re}\left[\phi''(z) + \bar{z}\,\psi''(z) + 2\,\psi'(z)\right]\right\} \qquad (175)$$

$$\frac{\partial^2 w}{\partial y^2} = -\left(\frac{\partial^2 w}{\partial z^2} - 2\frac{\partial^2 w}{\partial z\,\partial \bar{z}} + \frac{\partial^2 w}{\partial \bar{z}^2}\right) = \frac{-P}{16\,\pi\,N}\left\{\frac{z^2 + z^2}{z\,\bar{z}} - (4 + 2\log z\,\bar{z}) + 2\,\mathrm{Re}\left[\phi''(z) + \bar{z}\,\psi''(z) - 2\,\psi'(z)\right]\right\} \qquad (176)$$

$$\nabla^2 w = \frac{4\,\partial^2 w}{\partial z\,\partial \bar{z}} = \frac{P}{4\,\pi\,N}\left[2 + \log z\,\bar{z} + \psi'(z) + \overline{\psi'(z)}\right] \quad (177)$$

$$\frac{\partial^2 w}{\partial x\,\partial y} = i\left(\frac{\partial^2 w}{\partial z^2} - \frac{\partial^2 w}{\partial \bar{z}^2}\right)$$

$$= \frac{i\,P}{16\,\pi\,N}\left\{\frac{z^2 - \bar{z}^2}{z\,\bar{z}} + \mathrm{Im}\left[\phi''(z) + \bar{z}\,\psi''(z)\right]\right\} \quad (178)$$

$$\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2} = 2\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial \bar{z}^2}\right) = \frac{P}{16\,\pi\,N}\left[8 + 4\log z\,\bar{z} + 8\,\mathrm{Re}\,\psi'(z)\right] \quad (179)$$

$$M_x = -N\left(\frac{\partial^2 w}{\partial x^2} + \nu\,\frac{\partial^2 w}{\partial y^2}\right) = -N\,\nabla^2 w + N\,(1 - \nu)\,\frac{\partial^2 w}{\partial y^2} \quad (180)$$

$$M_y = -N\left(\frac{\partial^2 w}{\partial y^2} + \nu\,\frac{\partial^2 w}{\partial x^2}\right) = -N\,\nabla^2 w + N\,(1 - \nu)\,\frac{\partial^2 w}{\partial x^2} \quad (181)$$

$$V_x = -N\,\frac{\partial}{\partial x}\,\nabla^2 w = \frac{-P}{4\,\pi}\left[\frac{z + \bar{z}}{z\,\bar{z}} + \psi''(z) + \overline{\psi''(z)}\right] \quad (182)$$

$$V_y = -N\,\frac{\partial}{\partial y}\,\nabla^2 w = \frac{-i\,P}{4\,\pi}\left[\frac{\bar{z} - z}{z\,\bar{z}} + \psi''(z) - \overline{\psi''(z)}\right] \quad (183)$$

$$M' = \frac{-N}{2}\,(1 + \nu)\,\nabla^2 w = -N\,(1 + \nu)\,2\,\frac{\partial^2 w}{\partial z\,\partial \bar{z}} \quad (184)$$

$$M'' = \frac{-N}{2}\,(1 - \nu)\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2}\right) = -N\,(1 - \nu)\left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial \bar{z}^2}\right) \quad (185)$$

$$M_x = M' + M'' \quad (186)$$

$$M_y = M' - M'' \quad (187)$$

In Eqs. 174 to 187, $\psi'(z)$ and $\overline{\phi'(z)}$ denote $\frac{\partial \psi}{\partial z}$, $\frac{\partial \phi}{\partial \bar{z}}$, etc.; Re means "real part" and Im means "imaginary part." All the right members of these equations are real.

Example 4. Clamped Circular Plate with Central Point Load.—The point load P is located at the center of a clamped plate of radius a. If the edge of the plate is clamped horizontally at $x^2 + y^2 = a^2$, then the right member of Eq. 174 vanishes when $z\,\bar{z} = a^2$, and

$$z\,(1 + \log z\,\bar{z}) + \psi(z) + z\,\psi'(z) + \overline{\phi'(z)} = 0 \quad (188)$$

with $z = a\,e^{i\theta}$. Since $\phi(z)$ and $\psi(z)$ are analytic within $|z| \eqslantless a$, they may be represented as simple power series:

$$\phi(z) = b_o + b_1\,z + b_2\,z^2 + \cdots \quad (189a)$$

and

$$\psi(z) = c_o + c_1\,z + c_2\,z^2 + \cdots \quad (189b)$$

Inserting Eqs. 189 in Eq. 188 for boundary values of z, one readily determines:

$$c_1 = -\tfrac{1}{2}(1 + 2\log a), \qquad b_1 = -c_o \ldots\ldots\ldots\ldots (190a)$$

$$c_n = b_n = 0, \quad \text{for} \quad n \geqq 2 \ldots\ldots\ldots\ldots\ldots\ldots (190b)$$

and

$$w(r) = \frac{P}{8\,\pi\,N}\left(r^2 \log\frac{r}{a} + b_o - \frac{r^2}{2}\right) \ldots\ldots\ldots\ldots (190c)$$

Eq. 190*c* is obtained from Eq. 173 and the constant b_o will be $\frac{a^2}{2}$ if the deflection is measured from the plane of the clamped edge—that is, $w = 0$ at $r = a$. With this value of b_o, Eq. 190*c* is the required solution, and the place coefficients are $B = -\tfrac{1}{2}$ and $C = D = 0$.

Example 5. Circular Plate Centrally Loaded and Naviér Edge Conditions.—This edge condition requires that $\nabla^2 w = 0$ at $z\,\bar{z} = a^2$. The right member of Eq. 177 vanishes where $z = a\,e^{i\theta}$; that is,

$$2 + \log z\,\bar{z} + \psi'(z) + \overline{\psi'(z)} = 0 \ldots\ldots\ldots\ldots\ldots (191)$$

The coefficients in Eq. 189*b* become $c_1 = -(1 + \log a)$ and $c_n = 0$ for $n \geqq 2$. The property of radial symmetry requires that $c_o = 0$ and $b_n = 0$ for $n \geqq 1$, and from the deflection at the edge, $b_o = a^2$; hence,

$$w = \frac{P}{8\,\pi\,N}\left(r^2 \log\frac{r}{a} + a^2 - r^2\right) \ldots\ldots\ldots\ldots\ldots (192)$$

The place coefficients are $B = C = D = 0$. The physical meaning of the imposed boundary condition is that the moment sum vanishes at $r = a$; that is,

$$M_x + M_y = M_r + M_\theta = 0 \ldots\ldots\ldots\ldots\ldots\ldots (193)$$

This can be realized in a circular plate by clamping the edge so that the slope is given by

$$\frac{dw}{dr} = -\frac{P\,a}{4\,\pi\,N} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (194)$$

at the boundary. In a polygonal plate where the edges suffer no deflection this condition is equivalent to simply supported edges.

Example 6. Simply Supported Circular Plate Centrally Loaded.—The edge condition requires that

$$M_r = -N\left[\nabla^2 w - (1 - \nu)\frac{dw}{r\,dr}\right]$$

$$= -N\left[\nabla^2 w - (1 - \nu)\left(\frac{1}{\bar{z}}\frac{\partial w}{\partial z} + \frac{1}{z}\frac{\partial w}{\partial \bar{z}}\right)\right] = 0 \ldots\ldots\ldots (195)$$

at $z\,\bar{z} = a^2$. One may use this operator in Eq. 173 and evaluate it at the boundary. Symmetry conditions and the vanishing of the boundary deflec-

tion lead to

$$w = \frac{P}{8\,\pi\,N}\left[\,r^2 \log\frac{r}{a} + \frac{3+\nu}{2\,(1+\nu)}\,(a^2 - r^2)\,\right]\ldots\ldots\ldots\ldots(196)$$

It is natural for the reader to inquire as to what has been gained by using two unknown analytic functions. The answer is not immediately given. The simplicity of Examples 4, 5, and 6 is such that the method of this analysis seems unnecessarily cumbersome, since other more direct methods are available. In the following examples, in which the radial symmetry is not present, the writer uses the method of mapping, conformally, the area of the given plate. This plate is considered to occupy a region of the complex z-plane which is mapped into a region of the complex ζ-plane, which is entirely within a circle of unit radius. The boundary of the plate in the z-plane becomes the boundary of the unit circle $|\zeta| = 1$ in the new plane.

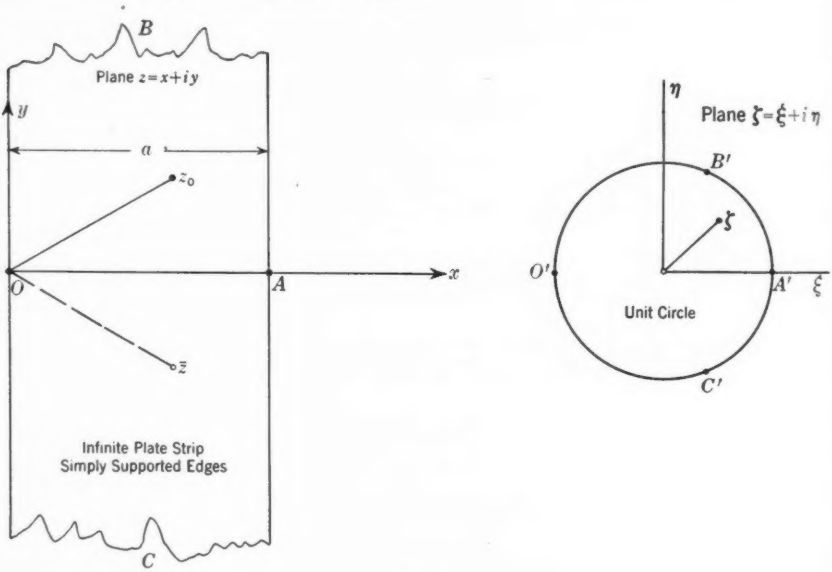

FIG. 5

Examples of Mapping to a Unit Circle.—Let $z = z(\zeta)$ be a mapping transformation by which a 1 : 1 correspondence is established between the boundary points of the edge of the plate and the point on the periphery of the unit circle in the ζ $(= \xi + i\,\eta)$-plane. Furthermore, let the transformation take the load point $z = z_o$ into the origin $\zeta = 0$ of the circle.

First, one may treat the case of a polygonal plate with rectilinear edges which are simply supported so that the boundary condition is $\nabla^2 w = 0$. The moment sum $M' = \dfrac{M_x + M_y}{2}$ is proportional to $\nabla^2 w$ and hence satisfies $\nabla^2 M' = 0$ at all points in the plate except at the point where the concentrated load is located. The right member of Eq. 177 shows that M' has a singularity

of the type $\log z\,\bar{z}$. It is known from the theory of logarithmic potential[46] that the function M' can be determined from

$$M' = \kappa \log |\zeta| \qquad (197)$$

in which $\zeta = \zeta(z)$ is the aforementioned mapping function and κ is a constant to be determined to meet the singularity condition of a point load given by Eq. 158. Some examples of this method follow.

Example 7. Simply Supported Infinite Plate Strip; Point Load.—This is Example *c* of the paper.[47] The simply supported infinite plate strip of width a has the coordinate axis and load point z_o shown in Fig. 5. This figure also shows the unit circle in the ζ-plane and the correspondence of points O, C, A, and B on the boundary of the infinite plate with the points O′, C′, A′, and B′, respectively. The transformation which transacts this mapping in such a manner that z_o goes into the origin $\zeta = 0$ is:

$$\zeta = \rho\, e^{i\lambda} = \frac{\sin \frac{\pi}{2\,a}(z - z_o)}{\sin \frac{\pi}{2\,a}(z + \bar{z}_o)} \qquad (198)$$

The constant κ in Eq. 197 can be determined so that

$$N\,\nabla^2 w = \frac{P}{2\,\pi}\log\rho = \frac{P}{2\,\pi}\,\mathrm{Re}\left[\log\sin\frac{\pi}{2\,a}(z - z_o) - \log\sin\frac{\pi}{2\,a}(z + \bar{z}_o)\right]$$

$$N\,\nabla^2 w = \frac{P}{4\,\pi}\log\frac{\cosh\frac{\pi}{a}(y - y_o) - \cos\frac{\pi}{a}(x - x_o)}{\cosh\frac{\pi}{a}(y - y_o) - \cos\frac{\pi}{a}(x + x_o)} \qquad (199)$$

For small values of $|z - z_o|$ this leads to the place coefficients

$$C = 1, \qquad D = 0, \qquad \text{and} \qquad B = \log\frac{2}{\pi}\sin\frac{\pi\,x_o}{a} \qquad (200)$$

Example 8. Simply Supported Half Infinite Strip.—If one half of the strip in Fig. 5 is a simply supported plate, the region O A B can be mapped upon a unit circle in the ζ-plane by the transformation:

$$\zeta = \rho\, e^{i\lambda} = \frac{\sin\frac{\pi}{2\,a}(z - z_o)}{\sin\frac{\pi}{2\,a}(z + \bar{z}_o)} \times \frac{\sin\frac{\pi}{2\,a}(z + z_o)}{\sin\frac{\pi}{2\,a}(z - z_o)} \qquad (201)$$

and

$$N\,\nabla^2 w = \frac{P}{2\,\pi}\log\rho \qquad (202)$$

in which $\log\rho$ in Eq. 202 will contain two expressions similar to the one obtained from Eq. 198. The physical interpretation of the result is that the moment

[46] "Foundations of Potential Theory," by Oliver Dimon Kellogg (Julius Springer, Berlin), 1929, p. 365.
[47] See also, "Die elastischen Platten," by A. Nádai (Julius Springer, Berlin), 1925.

sum at any point in the simply supported half strip is the combination of two equal and oppositely directed loads applied to an infinite plate strip at the two points z_o and $\bar{z}_o$. In a small neighborhood of the load point $z_o = x_o + i\,y_o$,

$$N\,\nabla^2 w = \frac{P}{2\,\pi}\left\{\log\frac{|z - z_o|}{a} - \left[\log\left(\frac{2}{\pi}\sin\frac{\pi\,x_o}{a}\right) + 2\log\frac{\cosh\frac{2\,\pi\,y_o}{a} - 1}{\cosh\frac{2\,\pi\,y_o}{a} - \cos\frac{2\,\pi\,x_o}{a}}\right]\right\} \qquad (203)$$

The place coefficient B is the expression enclosed in the brackets.

Example 9. Sector of a Circle.—Consider a plate sector of a circle of angular opening $\alpha = \frac{\pi}{k}$, in which $k \gtrless 1$, simply supported along two radial edges which extend indefinitely. Let $z_o = r_o\,e^{i\theta_o}$ be the point of loading. The transformation,

$$\zeta = \frac{z^k{}_o - z^k}{z^k - \bar{z}^k{}_o} = \rho\,e^{i\lambda} \qquad (204)$$

maps the region of the plate sector within the unit circle with z_o going into $\zeta = 0$. If $R = \frac{r}{r_o}$, in which r and r_o are distances from the vertex of the plate sector, then

$$N\,\nabla^2 w = \frac{P}{4\,\pi}\log\frac{R^k + R^{-k} - 2\cos k\,(\theta_1 - \theta)}{R^k + R^{-k} - 2\cos k\,(\theta_1 + \theta)} \qquad (205)$$

If the load point is on the axis of symmetry and $\theta_1 = \frac{\alpha}{2}$, then

$$N\,\nabla^2 w = \frac{P}{4\,\pi}\log\frac{R^k + R^{-k} - 2\sin k\,\theta}{R^k + R^{-k} + 2\sin k\,\theta} \qquad (206)$$

If $\theta = \frac{\alpha}{2}$ and $\alpha = \frac{\pi}{2}$, then $k = 2$, and along the bisector of the plate,

$$N\,\nabla^2 w = \frac{P}{4\,\pi}\log\frac{r^4 + r^4{}_o - 2\,r^2\,r^2{}_o}{r^4 + r^4{}_o + 2\,r^2\,r^2{}_o} = \frac{P}{2\,\pi}\log\frac{|r^2 - r^2{}_o|}{r^2 + r^2{}_o} \qquad (207)$$

For $|r - r_o|$, which is a small value:

$$N\,\nabla^2 w = \frac{P}{2\,\pi}\log\frac{|r - r_o|}{r_o} \qquad (208)$$

This yields the place coefficient $B = 0.0$ when the sector is an infinite quadrant $\alpha = \frac{\pi}{2}$ and the load is at a distance r_o along the bisector. With $M_x = M_y$, $C = 1$ and $D = 0$. If the log term is put in the form, $\log\frac{R}{c}$, in which $c = 2\,r_o$

(that is, twice its distance to the nearest supporting edge), then $B = -\log \sqrt{2} = -0.347$. If $k = 1$, and $\theta = \theta_1 = \frac{\pi}{2}$, then Eq. 205 yields:

$$N \nabla^2 w = \frac{P}{2\pi}\left(\log \frac{|r - r_o|}{r_o} - \log 2\right) \qquad (209)$$

and $B = 0.693$ is the place coefficient for a point load at a distance r_o from a long, simply supported, rectilinear edge. It is to be noted that the values of B in these sector plates were derived by choosing a particular value of θ—namely, that along the bisector. In the special case $\alpha = \pi$, a simple particular transformation,

$$\zeta = \frac{i r_o - z}{i r_o + z} \qquad (210)$$

transforms the half plane with the point $z = i r_o$ going into the center of the ζ-circle. Hence,

$$N \nabla^2 w = \frac{P}{2\pi} \log \frac{\rho_1}{\rho_2} = \frac{P}{2\pi}\left(\log \frac{\rho_1}{r_o} - \log \frac{\rho_2}{r_o}\right) \qquad (211)$$

holds for any point in the plate where ρ_1 and ρ_2 are distances from the load point and its image, respectively. In this general case the place coefficient is $B = \log \frac{\rho_2}{r_o}$.

Example 10. Clamped Circular Plate with Eccentric Point Load.—A circle of radius a is mapped within the circle $|\zeta| = 1$ by the transformation,

$$\zeta = \frac{a(z - z_o)}{a^2 - z \bar{z}_o} \qquad (212)$$

with the load point, z_o, becoming the origin, $\zeta = 0$. No loss of generality is incurred if z_o is taken at a distance c along the real axis x, the value of c being the distance the load is from the center of the circular plate of radius a. Then $\log |\zeta|$ is $\log \frac{a \rho_1}{c \rho_2}$, in which ρ_1 and ρ_2 are the respective distances from any point of the plate to the load points and to its circular image, which is located on the real axis at a distance $\frac{a^2}{c}$ from the origin. Before considering the clamped edge, it may be pointed out that the latter logarithmic term determines the place coefficient at any point in an eccentrically loaded circular plate having a Naviér edge condition.

In the case of a clamped edge along which the deflection and slope vanish, a suitable deflection function is:

$$w = \frac{P}{8\pi N}\left[\rho^2{}_1 \log \frac{a \rho_1}{c \rho_2} + \psi(\rho_1) + \phi(\rho_2)\right] \qquad (213)$$

since it is known that $\log \frac{a \rho_1}{c \rho_2}$ vanishes for points along the boundary. The vanishing of the slope normal to the boundary edge is equivalent to the vanish-

ing $\frac{\partial w}{\partial \rho_1}$ and $\frac{\partial w}{\partial \rho_2}$. If $\psi(\rho_1)$ and $\phi(\rho_2)$ are simple polynomial expressions,

$$\left.\begin{aligned}\psi(\rho_1) &= b_o + b_1\,\rho_1 + b_2\,\rho^2{}_2 + \cdots\\ \phi(\rho_2) &= c_o + c_1\,\rho_1 + c_2\,\rho^2{}_2 + \cdots\end{aligned}\right\}\quad\text{(214)}$$

Then the complete solution of Michell's clamped circular plate eccentrically loaded is:

$$w = \frac{P}{8\,\pi\,N}\left[\rho^2{}_1 \log\frac{a\,\rho_1}{c\,\rho_2} + \frac{1}{2}\left(\frac{c^2}{a^2}\,\rho^2{}_2 - \rho^2{}_1\right)\right]\quad\text{(215)}$$

and

$$N\,\nabla^2\,w = \frac{P}{8\,\pi}\left[4\log\frac{a\,\rho_1}{c\,\rho_2} + 4 - \frac{4\,\rho_1}{\rho_2}\cos\beta + 2\left(\frac{c^2}{a^2} - 1\right)\right]$$

$$= \frac{P}{2\,\pi}\left[\log\frac{a\,\rho_1}{c\,\rho_2} + 1 - \frac{\rho_1}{\rho_2}\cos\beta + \frac{1}{2}\left(\frac{c^2}{a^2} - 1\right)\right]\quad\text{(216)}$$

From Eq. 216 one may find a place coefficient B. As a special case, if $c \to 0$ with $c\,\rho_2 = a^2$, and $\rho_2 \to \infty$, then $N\,\nabla^2\,w = \frac{P}{2\,\pi}\log\left(\frac{r}{a} + \frac{1}{2}\right)$, which agrees with the result obtained from Eq. 190*c*. Another special case arises if both c and a become infinite with $(a - c)$ remaining finite. Then,

$$N\,\nabla^2\,w = \frac{P}{2\,\pi}\left(\log\frac{\rho_1}{a_o} - \log\frac{\rho_2}{a_o} + 1 - \frac{\rho_1}{\rho_2}\cos\beta\right)\quad\text{(217)}$$

in which a_o now becomes the distance that the load is from a long, clamped, straight edge, and ρ_1 and ρ_2 are the distances from a point x,y of the plate to the load point and its image. The angle β is the angle between the radii ρ_1 and ρ_2.

C. A. P. Turner,[48] M. Am. Soc. C. E.[48a]—In a circular plate, supported at its edge, the twisting shears may be clockwise or counterclockwise—as a matter of chance. The contours developed will be the same, but the spiral of displacement is right-handed in one case and left-handed in the other.

With the square or column-supported slab, the sign, or the change of sign, of the twisting shears must be considered because it controls the travel of the load to the support. Its neglect makes the mathematical plate theories unsatisfactory in general, as stated by Sir William Thomson and P. G. Tait[49] with reference to their own work: "Mathematicians have not hitherto succeeded in solving this problem with complete generality, for any other form of plate than the circular ring (or circular disc with concentric circular aperture)." This criticism is applicable to the paper under discussion.

Scientific literature and technical discussions are encumbered with a mass of mathematical development, tending toward confusion instead of simplification of the subject matter in a practical manner. This results from academic efforts to develop unknown properties from a hypothetical premise, inconsistent with observed and known phenomena. The utility of mathematical theory depends upon whether the theory enables the engineer to form a more exact conception of the resistance of materials and of the science of design.

[48] Cons. Engr., Columbus, Ohio.

[48a] Received by the Secretary September 29, 1942.

[49] "Natural Philosophy," by William Thomson and P. G. Tait, 1867, Article 655, p. 504.

In 1906 the writer constructed the concrete frame and floors of the Somerset Block in Winnipeg, Manitoba, Canada, under Garson, Hodgins and Thompson, contractors. Ten or twelve weeks after casting the first floor, the writer met the contractors at the building and was told that an accident had furnished a great test of the strength of the floor construction. The slab was $5\frac{1}{2}$ in. thick, $18\frac{1}{2}$ ft square, and reinforced with $\frac{3}{8}$-in. round rods, 6 in. on centers through the middle third and 8 in. on centers for the remainder. A stone cornice block weighing approximately three fourths of a ton had slipped from the derrick boom and dropped 60 ft on to the center slab.

The writer was asked what damage he supposed had been done. He replied that an engineer would expect the stone to be broken and shattered no matter how it hit the slab. If a corner had struck the slab, there probably would have been a crease or indentation of $\frac{1}{8}$ in. where the corner struck, with no cracks or other damage to the slab. The astonished contractors believed that the writer must have been told previously of the details of the accident until assured otherwise. The writer questions whether the mathematics in the Westergaard paper would have assisted in predicting this result, because the author's mathematical theory is that of imaginary substances as explained by William J. Ibbetson.[50]

Although recognizing that resistance to elastic change in form was that of intermolecular forces, early elasticians assumed that these forces, although simple in action, were too complicated for mathematical treatment; so they adopted the Boscovitch infinitesimal point action of imaginary substances. Its major error lies in the assumption that the internal shift of energy equals the external work of the applied force, whereas the actual kinetic shift of internal energy is 100% greater than that assumed in mathematical theory.

After a lifetime of endeavor, Prof. Karl Pearson found himself so dissatisfied with the hypothetical character of mathematical elasticity[51] that, in evaluating the work of Kelvin, he expressed the belief that Kelvin's work in coordinating the temperature change under unit force with the coefficient of expansion, weight, mechanical equivalent, and specific heat would bring him recognition as the greatest elastician of his age. Notwithstanding this brilliant achievement, Kelvin so underestimated the mechanical equivalent of the temperature change that he attributed the increased elongation, due to heat radiation, to viscosity instead of to the natural result of energy shift.

Another fundamental error lies in the generalization[52] of Hooke's Latin expression:

> "Ut tensio sic vis: That is, The Power of any Spring is in the same proportion with the tension thereof. If one power stretch or bend it one space, two will bend it two, and three will bend it three, etc. Now as the Theory is very short, so the way of trying it is very easie."[53]

[50] "An Elementary Treatise on the Mathematical Theory of Perfectly Elastic Solids, with a Short Account of Viscous Fluids," by William J. Ibbetson, Macmillan and Co., London, England, and New York, N. Y., 1887, Chapter I.

[51] "A History of the Theory of Elasticity and of the Strength of Materials, from Galilei to the Present Time," by Isaac Toddhunter (edited by Karl Pearson), Cambridge Univ. Press, 1886–1893, Vol. IV.

[52] "A Treatise on the Mathematical Theory of Elasticity," by A. E. H. Love, Cambridge Univ. Press, 1927, p. 95.

[53] "Lectures de Potentia," by Robert Hooke, 1676.

In the thin metal of a spring, the increase of temperature in the compression side is instantly absorbed, through conduction, by the decrease in temperature in the tension side; and Hooke's law of the power of a spring is a law of nature quite different from that of axial force, which endows the specimen with a large increase in strength and resistance before the radiation is complete.

Thus, H. H. Campbell[54] found that the yield point of mild steel—39,000 lb per sq in., under a pulling speed of 0.1 in. per min—was increased to 45,000 lb per sq in. under a high speed of testing. Such differences in the yield point were actual and in no sense hypothetical.

How should elastic resistance be formulated if the textbook translation of Hooke's Latin does not fit the case? An axial force produces a change in volume proportional to the magnitude of the force within the natural limit of elasticity. The same force causes the same change in volume in a given metal, independent of its physical state—whether it is soft ingot, or is in a rolled or drawn condition—thereby coordinating the variations in Young's modulus, E, and Poisson's ratio, μ, which have puzzled philosophers for generations.

Twenty-one independent coefficients of elasticity have been developed in the mathematical elasticity of hypothetical substances, as opposed to the fact that there are no independent coefficients or properties in actual substances. All are interdependent in endless ways (see discussion on time, space, and energy by Arvid Reuterdahl[55]).

Quoting Messrs. Thomson and Tait:[56]

> "Although the problem of fulfilling arbitrary boundary conditions has not yet been solved for rectangular plates, there is one remarkable case of it which deserves particular notice; not only as interesting in itself, and important in practical application, but as curiously illustrating one of the most difficult points of the general theory."

This appears in the bending of a thin rectangular plate, with a thickness 150th of the span, supported at diagonally opposite corners and loaded at the other two. The inscribed square remains flat and the end triangles bend only in cylindrical curvature. A mathematical theory that deserves recognition should account for this phenomenon, but the mathematics presented by Dean Westergaard is incapable of explaining it.

From the days of Stahl, scholars have entertained fantastic notions of energy and matter, based upon figments of the mind unrelated to mechanical fact and observation. It took fifty years for the textbook writers to recognize the transformation of mechanical energy into heat, although demonstrated conclusively by Benjamin Thompson in 1799. By boring copper with a dull tool under water, he brought the water to the boiling point. A similar backwardness is exhibited in the textbooks of today in failing to recognize the elementary relation of energy to mass. Thus, the cubic energy of elastic resistance is mistreated as linear energy in unsatisfactory mathematical treatments such as those presented by current writers on the subject.

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[54] "Manufacture and Properties of Structural Steel," by H. H. Campbell, Ed. 1, Hill Publishing Co., New York, N. Y., and London, England, 1896.

[55] "Kinematics," by Arvid Reuterdahl.

[56] "Natural Philosophy," by William Thomson and P. G. Tait, 1867, Article 656, p. 505.

In a current text,[57] conservation of momentum is treated as if elastic resistance were linear energy instead of cubic, presenting the erroneous conclusion that the sum of the momenta of two elastic spheres is the same after impact as before; and the same metaphysical notion appears in the Germanic dream that the intra-atomic energy of a quarter gram of matter is equal to that which the practical engineer develops in burning 5,000 tons of coal.

Since it took old-time scholars fifty years to discard the German phlogistic hypothesis after the demonstration of the relationship of mechanical to heat energy by Thompson, the practical student wonders if present-day scholars will need more than fifty years to discard the complex, artificial, mathematical elasticity of imaginary substances in favor of the natural elasticity of actual substances—Young's modulus coordinated by arithmetic with the atomic weight and number, strength, specific heat, melting point, and the calories of chemical combination.

The development of the exact stress mechanism of plates is a much simpler procedure than the development of the exact stress mechanism of simple and continuous beams. This is true despite the fact that beam action seems simple as treated by the common theory of flexure until it is observed that the shear strain so computed may be from 30% to 130% greater than the actual when determined by precise methods.[58]

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[57] "Handbook of Chemistry and Physics"—Charles D. Hodgman, Editor—21st Ed., Chemical Rubber Co., Cleveland, Ohio, p. 1675.

[58] "Elasticity, Structure and Strength of Materials," by C. A. P. Turner, Pt. I, Minneapolis, Minn., 1939, pp. 260–292.

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DISCUSSIONS

PROFILE CURVES FOR OPEN-CHANNEL FLOW

Discussion

By Hunter Rouse, M. Am. Soc. C. E.

Hunter Rouse,[18] M. Am. Soc. C. E.[18a]—Hydraulic engineers many decades ago concluded that the mathematicians' analysis of fluid motion was quite without practical value. Although it is now gradually being conceded that mathematical hydrodynamics has many an engineering application, the tendency still remains in hydraulics to shun any analysis of a practical problem which has too much of a mathematical savor. The writer fears, therefore, that the hydraulician's reaction to this paper by a mathematician will follow the age-old argument: "Of what use mathematics when the variables themselves are not properly understood?"

The writer feels responsible to some extent for Professor Gunder's attack upon this problem, for it was during the writer's lectures on advanced fluid mechanics at Fort Collins, Colo., in July, 1940, that the author clarified the consistent failure of computed surface curves below the critical depth to agree with the traditional curves in the writer's text. Comments by the writer on the obvious difficulty of integrating the varied-flow equations then led Professor Gunder to perform this integration, largely to satisfy his own curiosity. The results were finally submitted to the Society, not—so far as the writer is aware—in the guise of a final solution to the varied-flow problem, but simply to place on record the information contained therein.

It is scarcely to be expected, of course, that a member of one profession should at once feel as much at home in another profession as in his own. It is the writer's experience, however, that mathematicians have far greater facility in grasping the salient features of an engineering problem than have engineers in mastering the mathematics necessary to its solution. For this reason, the writer most cordially welcomes Professor Gunder's initial contribution to the hydraulic engineering field, regardless of the merit of the paper itself. This profession, as already stated, still suffers from many misunderstandings of the

Note.—This paper by Dwight F. Gunder, Esq., was published in April, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1942, by Carl Rohwer, Assoc. M. Am. Soc. C. E.; September, 1942, by C. J. Posey, Assoc. M. Am. Soc. C. E.; and October, 1942, by Messrs. J. C. Stevens, and Boris A. Bakhmeteff and Nicholas V. Feodoroff.

[18] Prof., Fluid Mechanics, State Univ. of Iowa, and Assoc. Dir. in Charge of Laboratory, Iowa Inst. of Hydr. Research, Iowa City, Iowa.

[18a] Received by the Secretary October 22, 1942.

past century, and can only profit from a mathematician's ability to cope with functional relationships which are beyond the normal engineering grasp.

As to the actual merits of this paper, the writer believes that the conclusions would be far more significant if freed from the handicap of being based upon resistance formulas which are in themselves none too trustworthy. Although the Manning formula is not necessarily worse than those of Bazin and Kutter, as implied by the author in his second conclusion, it is obviously more convenient than the other two, else the author would not have chosen it for purposes of integration; that it yields results which differ from those of the other two is surely not sufficient reason to condemn it. On the other hand, it has been shown both experimentally and analytically that the actual resistance due to wall roughness varies with the logarithm, rather than some power, of the relative roughness. Since logarithmic and power functions can approximately coincide over only a limited range, the author's curves necessarily reflect the discrepancies to an ever-increasing degree as they approach an upper or lower limit.

To be explicit, for the case of flow in a very wide channel in which the resistance is independent of viscosity, the Chezy C may be expressed with very close approximation in the Kármán-Prandtl form:

$$\frac{C}{\sqrt{8\,g}} = 2.1 + 2 \log_{10} \frac{R}{k} \qquad (40)$$

in which the factor k is a linear measure of the boundary roughness. (It is

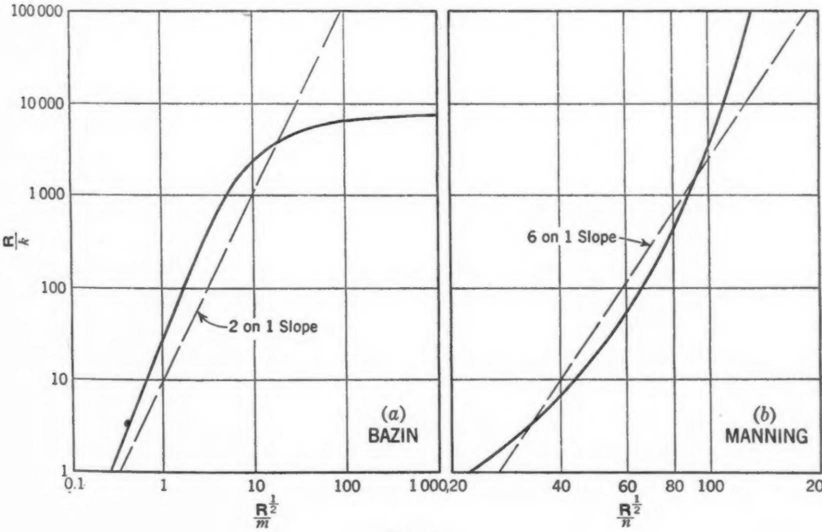

FIG. 8

to be noted that the dimensionless factor 2.1 is a function of cross-sectional form, and will vary with the proportions of the channel section.) Like R/k, the Bazin parameter $m/R^{1/2}$ and the Manning parameter $R^{1/6}/n$ are measures of

relative roughness, the Bazin m in American units obviously having the dimension of $(\text{feet})^{1/2}$, and the Manning n the dimension of $(\text{feet})^{1/6}$; what dimension to ascribe to the Kutter n is a problem, owing to the two different dimensions it appears to have in the Kutter formula. Were Eq. 40 and the Bazin formula both correct, a logarithmic plot of R/k against $R^{1/2}/m$ for identical values of C would appear as a straight line having a slope of 2 on 1; as seen from Fig. 8(a), the actual relationship has approximately the required slope only in the range of moderate to excessive roughness. Similarly, were Eq. 40 and the Manning formula in agreement, a similar plot of R/k against $R^{1/6}/n$ would yield a straight line having a 6 on 1 slope, whereas the curve in Fig. 8(b) displays this slope in only the intermediate zone, deviating therefrom as the roughness becomes either relatively great or relatively small. Only on this basis does the author's second conclusion seem justified.

Although Eq. 8 is undoubtedly more accurate over a wide range than any of the purely empirical formulas, it too becomes open to question as R/k approaches unity. Obviously, moreover, it is far less easy to adapt to an analysis such as the author's. Nevertheless, it is with considerable anticipation that the writer looks forward to Professor Gunder's incorporation of this relationship into his final verdict upon the shape of the surface profiles under discussion.

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DISCUSSIONS

MOMENT BALANCE: A SELF-CHECKING ANALYSIS OF RIGIDLY JOINTED FRAMES

Discussion

By Messrs. Bruce Jameyson, R. E. Bowles, and William Morris

Bruce Jameyson,[16] Assoc. M. Am. Soc. C. E.[16a]—To evaluate properly the relative merits of the new Cornish Method of Moment Balance the fact should not be overlooked that the Cross Method of Moment Distribution will accomplish nearly the same results and, in addition, as stated by the author, will include the sidesway feature more readily for complicated frames. When starting with the correct answers, moment distribution will balance most types of problems in one cycle, thereby partly checking these answers. If the answers are incorrect, one cycle will so indicate and usually will show the approximate location of the error or errors. Errors will be corrected in one or more cycles. If fair starting values are assumed, convergence is rapid. The accuracy (decimal places) of the trial answers may be increased. The foregoing statements apply equally well when sidesway is considered. To illustrate, the author's example is worked by moment distribution, using his starting trial answers.

Example.—Consider the example and frame shown in Fig. 2. The computations in Table 6 are almost self-explanatory. Joint rotation moments (M_R-values) were computed for the six upper joints, using the author's assumed starting answers in the girders and fixed-end moments (F-values). For example, for joints A and B:

$$M_{AB} + \tfrac{1}{2}\, M_{BA} = \text{answer} - F = 3.0 - 8.44$$

and

$$\tfrac{1}{2}\, M_{AB} + M_{BA} = \text{answer} - F = 2.5 - (-2.81)$$

Note.—This paper by R. J. Cornish, Esq., was published in May, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1942, by Messrs. D. D. Matthews, and William A. Larsen; June, 1942, by Messrs. A. A. Eremin, L. E. Grinter, and B. J. Aleck; September, 1942, by Messrs. Frederick S. Merritt, and Ralph W. Stewart; and October, 1942, by G. W. Stokes, Esq.

[16] Associate Prof., Civ. Eng., Univ. of California, Berkeley, Calif.

[16a] Received by the Secretary August 11, 1942.

Solving, joint rotation moments (M_R-values) are:

$$M_{AB} = -7.46$$

and

$$M_{BA} = 4.04$$

These two moments are the controlling moments for rotation of joints A and B, respectively. Then

TABLE 6.—Com-

Description	Joint A			Joint B				Joint C		
Member	....	AD	AB	BA	BE	....	BC	CB	CF	....
Stiffness factors, K	....	3	5	5	4	....	5	5	3	....
Distribution	....	0.375	0.625	0.357	0.286	....	0.357	0.625	0.375	....
Carry-over factors, C	....	↓ 0.5	0.5←	→0.5	↓ 0.5	....	0.5←	→0.5	↓ 0.5	....
Estimated answers, author's values	....	−3.0	+3.0	−2.5	+1.0	....	+1.5	0	0	....
(a) First Cycle										
Fixed-end moments, F	....	0	+8.44	−2.81	0	....	0	0	0	....
Computed joint rotations, M_R	....	−4.48	−7.46[a]	+4.04[a]	+3.23	....	+4.04	−1.00[a]	−0.60	....
Carry-over factors, C_R	....	+0.33	+2.02	−3.73	−0.92	....	−0.50	+2.02	+0.65	....
Sidesway corrections, f, neglected	....	....	....	....	....	....	....	....	....	....
Balancing moments, B	....	+0.43	+0.72	−1.20	−0.96	....	−1.19	−0.67	−0.40	....
(b) Second Cycle										
Carry-over factors, C	....	+0.08	−0.60	+0.36	−0.52	....	−0.33	−0.58	+0.14	....
Sidesway corrections, f, neglected	....	....	....	....	....	....	....	....	....	....
Balancing moments, B	....	+0.20	+0.32	+0.18	+0.14	....	+0.17	+0.28	+0.16	....
(c) Totals										
Answers at end of two cycles	....	−3.4	+3.4	−3.2	+1.0	....	+2.2	+0.05	−0.05	....
Close answers at end of four cycles	....	−3.42	+3.42	−3.10	+0.88	....	+2.22	0.00	0.00	....
Author's answers at end of two cycles	....	−3.5	+3.5	−3.0	+0.8	....	+2.2	−0.1	+0.1	....

$$M_{AD} = \frac{3}{5} \times (-7.46) = -4.48$$

$$M_{BE} = \frac{4}{5} \times 4.04 = 3.23$$

and

$$M_{BC} = \frac{5}{5} \times 4.04 = 4.04$$

In actual practice (except when checking answers, say, from another method) these joint rotations would not be computed, as in Table 6, but would be guessed directly instead of guessing the trial answers. Some work thus

would be saved. Also, little would be gained by attempting to guess joint rotations at joints C and D, which are far away from any load. This, then, becomes simply the general application of the arbitrary joint-rotation, short-cut method treated by Professor Cross and applied by him to a very special case.[17] The rotation moments may be applied to any or all joints, but, when the control rotation moment for one end of one member has been computed

PUTATION SHEET

Description	Joint D			Joint E				Joint F		
Member	DG	DA	DE	ED	EB	EH	EF	FE	FC	FK
Stiffness factors, K	4	3	6	6 ·	4	5	6	6	3	4
Distribution	0.308	0.231	0.461	0.286	0.190	0.238	0.286	0.461	0.231	0.308
Carry-over factors, C	↓ 0.5	↑ 0.5	0.5←	→0.5	↑ 0.5	↓ 0.5	0.5←	→0.5	↑ 0.5	↓ 0.5
Estimated answers, author's values	+2.0	−1.0	−1.0	−4.0	−3.0	−3.0	+10.0	−4.0	+2.0	+2.0
(a) First Cycle										
Fixed-end moments, F	0	0	0	0	0	0	+11.46	−5.21	0	0
Computed joint rotations, M_R	+0.89	+0.67	+1.33[a]	−2.76	−1.84	−2.30	−2.76[a]	+2.59[a]	+1.30	+1.73
Carry-over factors, C_R	0	−2.24	−1.38	+0.67	+1.62	0	+1.30	−1.38	−0.30	0
Sidesway corrections, f, neglected	....	....	....	....	....	....	....	....	....	....
Balancing moments, B	+0.22	+0.17	+0.34	−1.54	−1.03	−1.28	−1.54	+0.59	+0.29	+0.39
(b) Second Cycle										
Carry-over factors, C	0	+0.22	−0.77	+0.17	−0.48	0	+0.30	−0.77	−0.20	0
Sidesway corrections, f, neglected	....	....	....	....	....	....	....	....	....	....
Balancing moments, B	+0.17	+0.13	+0.25	0	0	0	+0.01	+0.45	+0.22	+0.30
(c) Totals										
Answers at end of two cycles	+1.3	−1.05	−0.2	−3.5	−1.7	−3.6	+8.8	−3.7	+1.3	+2.4
Close answers at end of four cycles	+1.28	−0.98	−0.30	−3.46	−1.76	−3.66	+8.88	−3.78	+1.35	+2.43
Author's answers at end of two cycles	+1.3	−1.0	−0.3	−3.4	−1.8	−3.6	+8.8	−3.8	+1.4	+2.4

[a] Controlling moment for joint.

or assumed, the M_R-values for all other members framing into the joint must be in accordance with relative stiffnesses (K-values). This follows from the fundamental definition of stiffness—that is: "The moment at one end of a member necessary to produce unit rotation of the same end, opposite end fixed (or free) and with no relative end deflection." Carry-over factors from these rotation moments (C_R) must be applied to opposite ends of members thus rotated. Note that, for member BA, $F + M_R + C_R$ = assumed estimated starting answer. Table 6 shows "Close answers at end of four cycles" (work

[17] "Continuous Frames of Re-inforced Concrete," by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1932, pp. 228, 229, and 230.

of third and fourth cycles not shown). These answers are correct to a maximum possible error of 0.02.

The following facts may be noted:

(*a*) At the end of the first cycle results by moment balance are more accurate than by moment distribution.

(*b*) At the end of the second cycle both methods yield results of about the same accuracy (nine differences of 0.1 in each case).

(*c*) Original starting answers (moments) have been roughly checked, and errors and mistakes in these starting answers have been corrected.

(*d*) The computations (except the original answers) were carried by slide rule to two decimal places, thereby increasing the accuracy of the four-cycle answers. Some of the computations by moment balance could not have been obtained to this accuracy by slide rule.

(*e*) One cycle of moment distribution, other than the first, involves smaller numerals and less work than for moment balance. The first cycle of both methods involves roughly the same amount of work when M_R-values are guessed, in moment distribution, instead of the answers.

(*f*) In this problem, sidesway corrections (fixed-end moments f), if desired, would be added directly after the carry-over values and would be based upon the subtotals at this stage.[18] In either case, if the starting answers were correct, all balancing moments would be zero, and the first and only cycle would partly check the assumed answers. The f-values, for this problem, are so easily computed that there usually would be little excuse for neglecting sidesway.

Conclusion.—It is the writer's opinion that the advantage is with moment distribution, at least in so far as the solution of the author's particular example is concerned. The writer believes, however, that the author has made a valuable contribution and that his method will find a permanent place in the field of structural analysis.

R. E. Bowles,[19] Esq.[19a]—The theory upon which this paper is based is a simple one and one that has now probably become familiar to those interested. It does not involve any intermediate assumptions that are not in accordance with the actual structure. Thus it avoids the complication of locking and releasing the joints. The paper is a valuable contribution to the subject for the following reasons:

1. The process solves, directly, for the quantity required—that is, the moments, and not the slopes.

2. It is simple in operation, largely self-checking, and error-eliminating.

3. It is considered to be an advantage that the resulting calculations are not written on the frame diagram, but are written in a table considerably easier to arrange than that of moment distribution for a complicated structure. It

[18] "Lateral Loads and Members of Variable Section," by G. E. Lange and C. T. Morris, *Bulletin No. 66*, Ohio State Univ. Studies, Engineering Series, 1931.

[19] Dept. of Building, College of Technology, Manchester, England.

[19a] Received by the Secretary September 14, 1942.

avoids the addition of the results of successive stages, which in moment distribution involves the summation of numerous quantities of opposite sign.

4. As the order of the "balances" is unimportant it is possible to concentrate on those parts of the structure where given loading produces maximum effect.

5. It permits the experienced designer to minimize work by inserting intelligent forecasts of the desired results, whereas less experienced designers may obtain equally accurate results with slightly more labor.

6. A few simple examples that have been solved by this method suggest that even without experience the designer finds it less laborious and quicker than other methods.

7. The practical user will probably appreciate that at each successive stage the required moment is seen clearly without resort to summation.

8. Although the method, as published, does not directly accommodate varying moments of inertia or sidesway, it may still be applied to many practical problems where the effects of these factors are frequently neglected.

The foregoing conclusions have been confirmed mostly by actual computations for a simple frame, which the writer had previously solved, for comparison, by other methods.

William Morris,[20] Esq.[20a]—Mr. Cornish appears to be justified in his claim to have established a method which is self-checking, and which gives quite a rapid solution of rigid frames. The writer tested the method on several frames, using assumed end moments which were obviously a bad guess, and even used moments of the wrong sign. It was found that three balances gave results which were sufficiently accurate in the worst case of assumption.

It does not seem necessary, however, to calculate the values of X by use of the formulas given in Table 1, as the fixing moments can be used directly for this purpose. To avoid the excessive amount of arithmetic which appears to be involved, the writer suggests the following form of solution which will, no doubt, commend "moment balance" to those engineers who are familiar with the moment distribution form of solution. For example, Eq. 5 may be written in the form

$$M_{ON} = \frac{1}{2} M_{NO} - \frac{K_{ON}}{\Sigma K}\left[\sum\left(-\frac{X}{2}\right) + \sum \frac{M'}{2}\right] + \frac{1}{2}(-X_{ON}) \ldots (20)$$

This indicates a process similar to moment distribution as follows:

(*a*) Assume moments at the ends of the members remote from the joint under consideration;

(*b*) Carry half these moments over to the joint;

(*c*) Sum the latter together with $\sum\left(-\frac{X}{2}\right)$ and distribute to each member in proportion to their K-values; and

(*d*) Add the results obtained (not forgetting to include $-\frac{X}{2}$ for those members which are loaded) to give the final moments. The process is continued

[20] With Matthews and Mumby, Manchester, England.

[20a] Received by the Secretary August 5, 1942.

around the frame as shown in the paper. It is to be noted that the values of $-\frac{X}{2}$ for each end of a loaded member can be obtained from the fixing moments by carrying over to the far end half the moment of the near end and changing sign. Sum the results to give $-\frac{X}{2}$ for each end.

It seems preferable to determine the values of $-\frac{X}{2}$ for all loaded spans first, although, for a building frame designed by loading only one span at a time to give maximum moments, two values only are required. This means that the values of $-\frac{X}{2}$ are only included in the balancing twice, as Eq. 20 reduces to the form

$$M_{ON} = \frac{1}{2} M_{NO} - \sum \frac{M'}{2} \qquad (21)$$

for the remaining joints.

The process is illustrated in Fig. 11 on which values of $-\frac{X}{2}$ have been kept clear of the calculations and the assumed moments have been written at

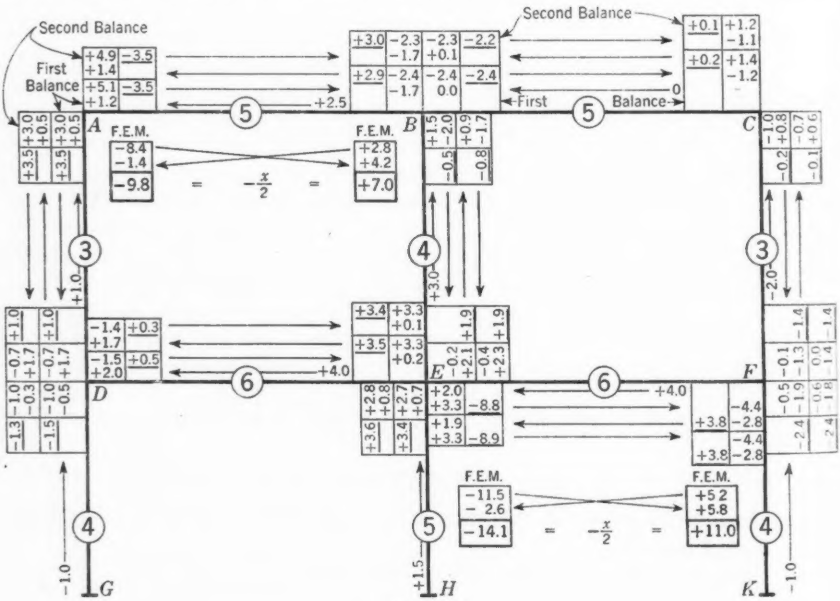

FIG. 11

the side of the table. The example chosen is that used by Mr. Cornish and the results at each joint are naturally identical with those of the author, who shows that they "self-check." The method is easily adaptable to portal frames without sidesway and continuous beams and, of course, can be used for haunched beams when the fixed-end moments and stiffness values have been ascertained.

The practical usefulness of the method for problems involving sidesway is not apparent.

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DISCUSSIONS

NUMERICAL PROCEDURE FOR COMPUTING DEFLECTIONS, MOMENTS, AND BUCKLING LOADS

Discussion

By Camillo Weiss, M. Am. Soc. C. E.

Camillo Weiss,[27] M. Am. Soc. C. E.[27a]—The method outlined will undoubtedly be found useful in many types of problems other than those discussed by Professor Newmark, and the determination of ordinates to influence lines is one of these. It is readily applicable because influence lines can be considered as ratios between corresponding deformations. Furthermore, because only ratios are required, the various "common factors" may be disregarded, and scales may be adopted and changed to suit convenience at any step in the consecutive computations, provided relative scales remain the same. The moment diagrams are bounded by straight lines; therefore the results are accurate for straight-line or parabolic variations of moments of inertia. For other variations satisfactory approximations may be obtained.

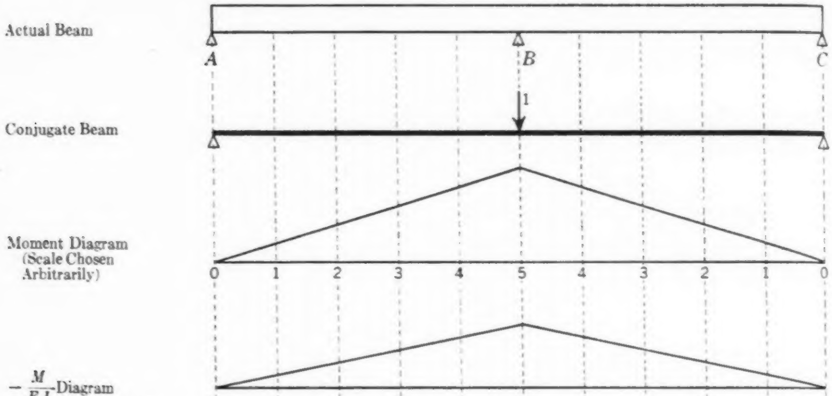
The writer has computed influence lines for three typical cases, shown in Figs. 33, 34, and 35, and a study of these calculations will show readily the relative ease of the work required. The conventional calculation methods involve the same steps, but by applying the author's method the amount of laborious arithmetical work is greatly reduced.

The influence diagram in Fig. 35 is obtained as the algebraic sum of two elastic curves of the simple beam AD. The first elastic curve is for a unit load at C acting downward. The second elastic curve is for a concentrated load at B chosen to nullify the deflection at B. This is accomplished by prorating the deflections due to a unit load at B acting upward. The procedure may be extended to any number of supports. It may be of interest to note that for a structure comparable to that of Fig. 35, but with vertical columns continuous at B and C and hinged at their bases, conventional methods

Note.—This paper by N. M. Newmark, Assoc. M. Am. Soc. C. E., was published in May, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1942, by Bruce Johnston, Assoc. M. Am. Soc. C. E.; June, 1942, by M. S. Ketchum, Jr., Assoc. M. Am. Soc. C. E.; September, 1942, by Messrs. John B. Wilbur, Ralph W. Stewart, and Stefan J. Fraenkel; and October, 1942, by Alfred S. Niles, Assoc. M. Am. Soc. C. E.

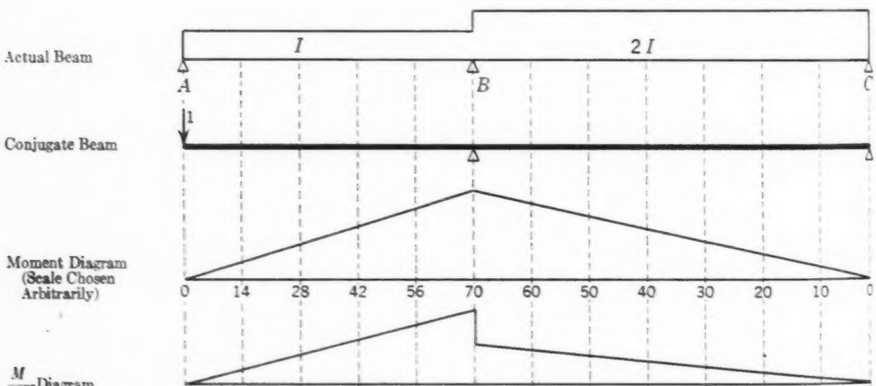
[27] Designer, Bethlehem Steel Co., Fabricated Steel Constr., Eng. Dept., Bethlehem, Pa.

[27a] Received by the Secretary October 5, 1942.



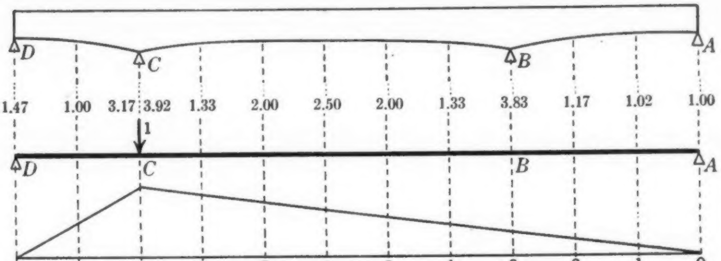
(Load on Conjugate Beam)	0	−1	−2	−3	−4	−5	−4	−3	−2	−1	0
Equivalent Concentrated Loads	−1	−6	−12	−18	−24	−28	−24	−18	−12	−6	−1
Assumed Average Slope (Assumed Symmetrical)		74	68	56	38	14	−14	−38	−56	−68	−74
Trial Deflection (= Actual Deflection)	0	74	142	198	236	250	236	198	142	74	0
Influence Line for Reaction at B	0	0.296	0.568	0.792	0.944	1.000	0.944	0.792	0.568	0.296	0

FIG. 33.—INFLUENCE LINE FOR REACTION AT CENTER SUPPORT OF BEAM CONTINUOUS OVER THREE SUPPORTS (SPANS EQUAL; MOMENT OF INERTIA CONSTANT)

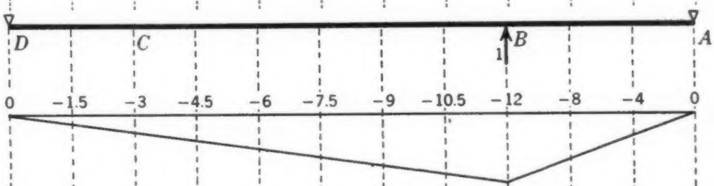


(Load on Conjugate Beam)	0	−14	−28	−42	−56	−70 ⋮ −35	−30	−25	−20	−15	−10	−5	0
Equivalent Concentrated Loads	−14	−84	−168	−252	−336	−296	−180	−150	−120	−90	−60	−30	−5
Assumed Average Slope		1,226	1,142	974	722	386	90	−90	−240	−360	−450	−510	−540
Trial Deflection	−4,450	−3,224	−2,082	−1,108	−386	0	90	0	−240	−600	−1,050	−1,560	−2,100
Linear Correction	−1,500	−1,200	−900	−600	−300	0	300	600	900	1,200	1,500	1,800	+2,100
Actual Deflection	−5,950	−4,424	−2,982	−1,708	−686	0	390	600	660	600	450	240	0
Influence Line for Reaction at A	1.000	0.744	0.501	0.287	0.115	0	−0.066	−0.101	−0.111	−0.101	−0.076	−0.040	0

FIG. 34.—INFLUENCE LINE FOR REACTION AT END SUPPORT A OF BEAM CONTINUOUS OVER THREE SUPPORTS (SPANS UNEQUAL; MOMENTS OF INERTIA DIFFER FOR SPANS BUT ARE CONSTANT WITHIN SPAN LENGTH)



	D		C						B			A
Relative Moments of Inertia	1.47	1.00	3.17 / 3.92	1.33	2.00	2.50	2.00	1.33	3.83	1.17	1.02	1.00
Moment Diagram (Scale Chosen Arbitrarily)	0	4.5	9	8	7	6	5	4	3	2	1	0
Ordinates of $(-)\frac{M}{EI}$ Diagram (Load on Conjugate Beam)	0	−4.50	−2.84 / −2.30	−6.0	−3.50	−2.40	−2.50	−3.01	−0.78	−1.71	−0.98	0
Equivalent Concentrated Loads	−4.5	−20.84	−20.79	−29.84	−22.41	−15.60	−15.41	−15.32	−7.84	−8.60	−5.63	−0.98
Assumed Average Slope		93.88	73.04	52.25	22.41	0	−15.60	−31.01	−46.33	−54.17	−62.77	−68.40
Trial Deflection	0	93.88	166.92	219.17	241.58	241.58	225.98	194.97	148.64	94.47	31.70	−36.70
Linear Correction	0	3.34	6.67	10.01	13.35	16.68	20.02	23.35	26.69	30.03	33.36	+36.70
Downward Deflection	0	97.22	173.59	229.18	254.93	258.26	246.00	218.32	175.33	124.50	65.06	0



	D		C						B			A
Moment Diagram (Note: Scale Must Be Same as for Moment Diagram Above)	0	−1.5	−3	−4.5	−6	−7.5	−9	−10.5	−12	−8	−4	0
Ordinates of $(-)\frac{M}{EI}$ Diagram (Load on Conjugate Beam)	0	1.50	0.95 / 0.77	3.38	3.00	3.00	4.50	7.90	3.13	6.84	3.92	0
Equivalent Concentrated Loads	1.50	6.95	8.32	17.29	18.38	19.50	28.90	39.23	27.26	34.41	22.52	3.92
Assumed Average Slope		−50.94	−43.99	−35.67	−18.38	0	+19.50	+48.40	+87.63	+114.89	+149.30	+171.82
Trial Deflection	0	−50.94	−94.93	−130.60	−148.98	−148.98	−129.48	−81.08	+6.55	+121.44	+270.74	+442.56
Linear Correction	0	−40.23	−80.47	−120.70	−160.93	−201.16	−241.40	−281.63	−321.86	−362.09	−402.33	−442.56
Upward Deflection	0	−91.17	−175.40	−251.30	−309.91	−350.14	−370.88	−362.71	−315.31	−240.65	−131.59	0
Prorated Upward Deflection (Ratio = 175.33 : 315.31)	0	−50.70	−97.53	−139.74	−172.33	−194.70	−206.23	−201.69	−175.33	−133.81	−73.17	0
Combined Elastic Curve (Downward Deflections + Prorated Upward Deflections)	0	46.52	76.06	89.44	82.60	63.56	39.77	16.63	0	−9.31	−8.11	0
Influence Line for Reaction at C	0	0.612	1.00	1.18	1.09	0.836	0.523	0.219	0	−0.122	−0.107	0

Fig. 35.—Influence Line for Reaction at Support C of a Beam Continuous Over Four Supports (Spans Unequal; Moments of Inertia Variable; the Variation Assumed To Be of Linear Dimension)

furnished the following influence ordinates: 0.000, + 0.617, + 1.00, + 1.16, + 1.04, + 0.775, + 0.448, + 0.161, 0.000, − 0.090, − 0.077, 0.000.

The constantly increasing number of indeterminate structures which are being built has made it essential for the designer to familiarize himself with the method of moment distribution devised by Hardy Cross,[28] M. Am. Soc. C. E. The author's method will provide great assistance in the determination of stiffness and carry-over factors, and in other less obvious ways.

Professor Newmark is to be congratulated for having produced a useful and well-presented paper, which is a definite contribution to engineering design methods.

Correction for *Transactions:* In May, 1942, *Proceedings*, page 713, Fig. 18(*d*), line 3, third column, change "344.5" to "334.5."

[28] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

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DISCUSSIONS

RELATIVE ANGULAR, LINEAR, AND TRAVERSE ACCURACIES IN CITY SURVEYS

Discussion

By G. Brooks Earnest, M. Am. Soc. C. E.

G. Brooks Earnest,[11] M. Am. Soc. C. E.[11a]—Far too often the question uppermost in the mind of the practicing, private surveyor is that of profit alone, rather than profit plus self-satisfaction resulting from assurance of relative accuracy of the individual surveys. Obviously, engineering economy dictates the essential equipment, together with the methods of operation and order of procedure for various classifications of surveys executed by the private surveyor. Therefore, specifications are usually neglected—which can be contributed largely to the existing multifarious public and private control surveys.

When one undertakes to make an exhaustive search of survey records through an accumulation of field books covering a practice of fifteen years or more, he is to be commended highly. Would that other practicing surveyors might similarly correlate their work, if for no other reason than to determine their budget of accuracies throughout stages of their practice.

In all probability, there would be little similarity between published accuracy curves of a score of territorily scattered, private surveyors. Many reasons contribute to this fact; such as: Instrumentation, individual (unique) methods of operation, topography, existence or nonexistence of precise control, taping procedure, and climate. Nevertheless, the underlying thought in such research is to inculcate "precision thinking" among those in the field of surveying—not with the intent that an estate survey in the suburbs should be executed with the same degree of precision as a "downtown" survey where property is worth several thousand dollars per front foot, but wholly with the idea of impressing the private surveyor with the value of uniformity of specifications for individual types of surveys.

Note.—This paper by S. A. Bauer, Assoc. M. Am. Soc. C. E., was published in May, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1942, by Messrs. Cleveland B. Coe, W. H. Rayner, Philip Kissam, and George D. Whitmore, C. C. Miner and W. O. Byrd.

[11] Asst. Prof., Civ. Eng., Case School of Applied Science, Cleveland, Ohio.

[11a] Received by the Secretary September 30, 1942.

The author's results represented by curves, (3) through (8) (Figs. 2, 3, and 4), are indicative of experienced personnel and painstaking field execution. However, to one who is experienced in traverse surveys of first-order and second-order nature and with the knowledge of the pitfalls encountered even with the use of 10″ repeating instruments, taping bucks, etc., to comply with the control precision specified, it is difficult to conceive the respectable percentage of high-order results from commonly called "third-order" methods of field operations. Of course, without precise control, loop closures must be relied upon to compute the probable error of closure, and there is the possibility of compensating errors fortunately producing occasional, exceptional order of precision.

Relative to angle observations and taped distances between two monuments of the common "garden variety" street intersection type, taken several years apart, there would naturally be no pattern of mean probable error (especially north of the Mason and Dixon Line). Furthermore, geological conditions, frost conditions, and depth of monument would affect such comparison measurements differently in cities widely separated in latitude. Such "lapse time-measurement" curves, therefore, conform strictly to individual conditions and are meaningless unless observations are between so-called permanent monuments that have their support below the frost line and are low in susceptibility to frost action.

Uniformity is the biggest factor in making related studies of the private, practicing surveyor. The Society's Surveying and Mapping Division recognized this fact when, in 1928, it formed the Committee on City Surveys and directed this Committee to draft a manual "for the purpose of describing the various divisions of the complete City Survey, to furnish specifications for it, and to make detailed recommendations as to accepted technical methods." The result of this Committee's study and research was published in 1934 as *Manual of Engineering Practice No. 10,* entitled "Technical Procedure for City Surveys." The scope of this manual dwelt chiefly on control surveys of highest order. Later, in 1940, due to increased importance in the subject, *Manual of Engineering Practice No. 20* was published entitled "Horizontal Control Surveys to Supplement the Fundamental Net." These manuals represent approximately ten years of active committee work in drafting the universally recommended procedures for the several orders of control. Therefore, it behooves the practicing surveyor, who desires to stay abreast of the field, to conform to procedures recommended in these manuals and thereby make possible tangible comparisons of individual accomplishments in the future.

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DISCUSSIONS

STATISTICAL ANALYSIS IN HYDROLOGY

Discussion

BY MESSRS. PAUL V. HODGES, AND JOE W. JOHNSON

PAUL V. HODGES,[17] M. AM. SOC. C. E.[17a]—That the straight-line logarithmic curves fit the rainfall records very well, and that the logarithms of the observations may be assumed to be normally distributed, are demonstrated in this paper. In Figs. 4(*b*) and 4(*d*), it appears, however, that curves developed by some other method, as for instance either the Slade,[3] Foster,[6] or Hazen[18] method, would give even closer agreement with the rainfall records.

Rainfall and runoff statistics for a period of record represent the samples for basing a prediction of what may be expected. The coefficient of variation and coefficient of skew of these samples represent their characteristics, although the coefficient of skew for available periods of record may be in error and therefore not truly representative.

To obtain a straight-line logarithmic curve it is often necessary to change both the coefficient of variation and coefficient of skew, and the question arises as to whether such a change would not change the characteristics of the sample so that it would fail to be truly representative.

In the development of curves by the Slade, Foster, and Hazen methods, it is often necessary to adjust the coefficient of skew to obtain a curve that best fits the data, but the computed coefficient of variation is a true representative of the sample as far as known, and should not be changed.

In Fig. 5 the yearly runoff of the Gila River at San Carlos Reservoir is shown. The coefficient of variation as computed for these data was 0.876. In drawing the curve for the Foster method this value of the coefficient of variation was used, but the coefficient of skew of 2.3 was found to give the best fit.

To obtain a straight line with the foregoing value of the skew coefficient,

NOTE.—This paper by L. R. Beard, Jun. Am. Soc. C. E., was published in September, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1942, by Messrs. L. Standish Hall, and H. Alden Foster.

[17] Hydr. Engr., U. S. Engrs. Office, Denison, Texas.

[17a] Received by the Secretary October 6, 1942.

[3] "An Asymmetric Probability Function," by J. J. Slade, Jr., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 35.

[6] *Ibid.*, Vol. 99 (1934) p. 1220.

[18] "Flood Flows," by Allen Hazen, John Wiley & Sons, Inc., 1930, p. 188.

a coefficient of variation of 0.82 must be used, according to the Hazen factors, thus changing the characteristics of the sample. Both curves appear to fit the data except for the lower values.

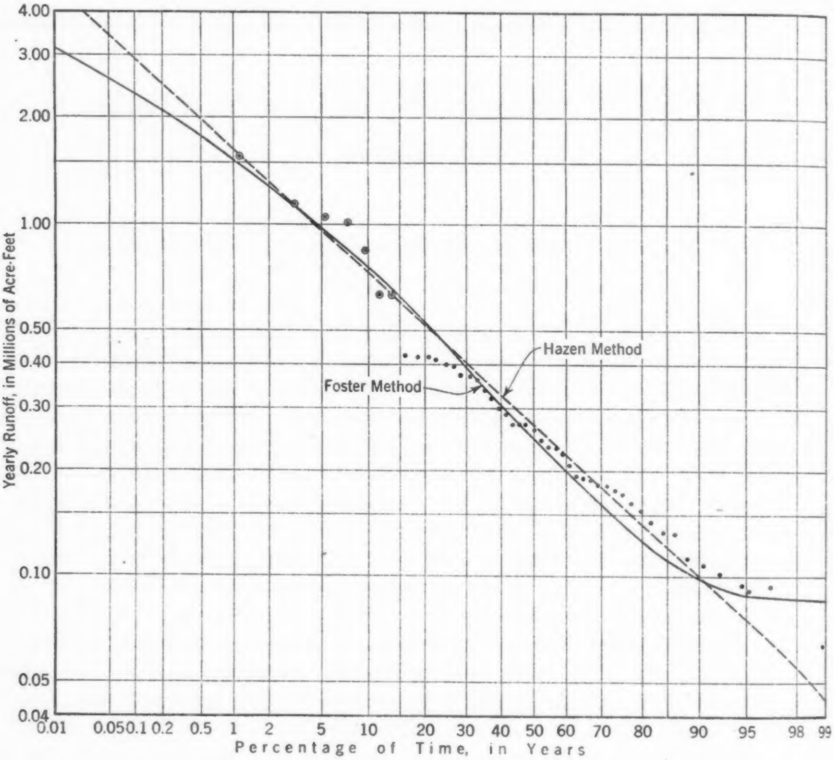

FIG. 5.—PROBABILITY CURVES FOR YEARLY RUNOFF IN ACRE-FEET, GILA RIVER AT SAN CARLOS RESERVOIR, ARIZONA, 1895–1941

A probability study of floods of the Red River near Denison, Tex., is shown in Fig. 6. Flows that averaged 30,000 cu ft per sec daily or more were selected from the tabulation of daily discharge and the mean flood was 72,400 cu ft per sec (maximum daily average), with a coefficient of variation of 0.664. During the period, 1906–1942, there have been 117 such occurrences. These floods occurred during different seasons of the year, the largest being in April, May, and June. They are considered to be a homogeneous series of flood quantities.

The curves in Fig. 6 were developed by the Slade and Foster methods and fit the data very much closer than any straight line.

In view of the foregoing examples it appears that rainfall, runoff, and flood occurrences may or may not indicate straight-line logarithmic curves, and that the various methods of developing curves are all valuable tools for use with

different statistical characteristics. It also appears that a straight-line logarithmic curve would be justified only in cases where it can be made to fit the data without changing the computed coefficient of variation.

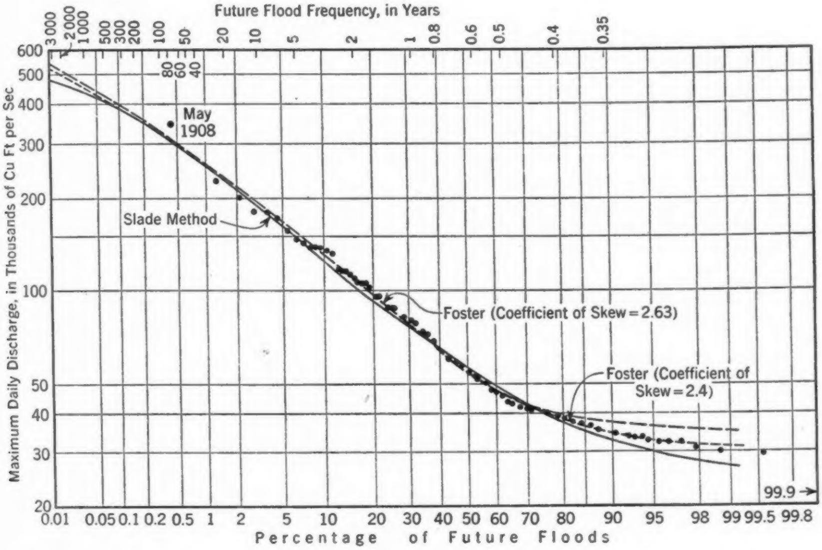

FIG. 6.—DENISON DAM AND RESERVOIR, PROBABILITY CURVES FOR MAXIMUM FLOODS (MAXIMUM DAY AVERAGE), 1906–1942

JOE W. JOHNSON,[19] ASSOC. M. AM. SOC. C. E.[19a]—In addition to flood-control problems, as discussed by the author, the application of the principles of the duration curve affords a valuable aid in estimating the average annual volume of bed load transported by streams. It is often the case, however, that estimates of bed-load transportation are required for streams for which discharge records are not available. A flow-duration curve, consequently, cannot be constructed for the stream in question, but it may be possible, in some instances, that the duration curve from an adjacent stream for which discharge records are available can be used. For example, Thorndike Saville, M. Am. Soc. C. E., and John D. Watson, Assoc. M. Am. Soc. C. E.,[20] found that, by using a ratio of the discharge to the mean discharge as the ordinate, the flow-duration curves of relatively large North Carolina streams could be represented by a single curve. To determine whether the same principles apply to streams of relatively small drainage where climate and topography are rather uniform and discharge records are expressed in daily averages (instead of weekly averages as in the streams studied by Professors Saville and Watson), a study was made of several small streams in the Piedmont region of North Carolina in conjunction with sediment-load investigations of the Soil

[19] Asst. Prof., Dept. of Mech. Eng., Univ. of California, Berkeley, Calif.

[19a] Received by the Secretary October 19, 1942.

[20] "An Investigation of the Flow-Duration Characteristics of North Carolina Streams," by Thorndike Saville and John D. Watson, *Transactions*, Am. Geophysical Union, 1933, pp. 406–425.

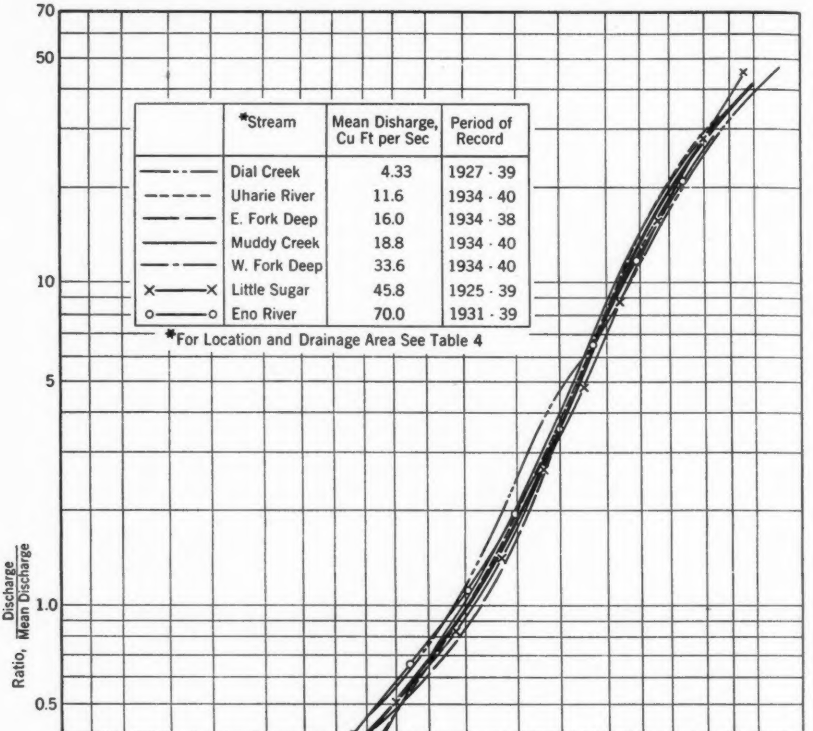

FIG. 7.—FLOW DURATION CURVES FOR SEVERAL SMALL STREAMS IN THE PIEDMONT REGION OF NORTH CAROLINA

Conservation Service.[21] The streams selected for study were of the size for which rates of bed-load transportation could be determined either by direct measurement or by application of a suitable formula. Discharge data used in the analysis were expressed in mean daily rates and were obtained from water supply papers of the U. S. Geological Survey.

Fig. 7 shows flow-duration curves for seven streams, with drainage areas ranging from 4.9 to 66.5 sq miles (see Table 4), plotted on probability paper. To eliminate the size of drainage area, the ratio of discharge to mean discharge has been used as the ordinate. Examination of these curves shows a remarkably close agreement for the upper 40% of the time—that is, for the higher rates of discharge, the various curves are almost identical. For that part of the curves representing the lower rates of flow, however, the curves depart radically from each other. These conditions are to be expected because, for low rates of flow, the drainage basin characteristics, such as soil types, vegetal cover, tillage methods, and stage of soil erosion, influence the rate of runoff greatly. However, during periods when high rates of precipitation cause high runoff rates, the drainage basin characteristics affect the runoff very little.

TABLE 4.—LEGEND FOR FIG. 8

Legend[a]	Stream	Location	Area[b]
	(a) PIEDMONT REGION OF NORTH CAROLINA		
①	Dial Creek	Bahama	4.9
●	Uharie River	Trinity	11.3
◆	East Fork, Deep River	High Point	13.9
◆	Muddy Creek	Archdale	16.2
✦	West Fork, Deep River	High Point	32.1
②	Little Sugar Creek	Charlotte	41.4
③	Eno River	Hillsboro	66.5
④	Fisher River	Dobson	109
⑤	Dan River	Francisco	119
⑥	Flat River	Bahama	150
⑦	Haw River	Benaja	168
	(b) IOWA AND MISSOURI		
■ ⑧	Ralston Creek	Iowa City, Ia.	3
■	West Tarkio Creek	Westboro, Mo.	200
■	Tarkio River	Blanchard, Ia.	105
	(c) MOUNTAINOUS REGIONS OF NORTH CAROLINA[c]		
○	Coweeta Creek 9	Asheville	2.79
ϕ	Coweeta Creek 8	Asheville	2.93
⊖ [1]	Beetree Creek	Swannanoa	5.46
✦ [2]	South Fork, Mills River	Pink Beds	9.99
[3]	Linville River	Branch	65
Δ	French Broad River	Rosman	67.9
[4]	Tye River	Roseland, Va.[c]	68
	(d) STATE OF WASHINGTON		
≜	Missouri Flats Creek	Pullman	27.5
≜	Four Mile Creek	Pullman	71.9
□	South Fork Palouse River	Pullman	81.1
	(e) WISCONSIN		
⊟	Little La Crosse River	Leon	77.1
ɸ	Coon Creek	Coon Valley	77.2

[a] Numerals in circles or squares denote a period of ten years or more. [b] Drainage area, in square miles. [c] All in North Carolina except Roseland (Va.).

Of interest in this respect is the comparison of the curves for the Uharie River and Muddy Creek with the curves for the East and West forks of Deep River (Fig. 7). The drainage basins of the Uharie River and Muddy Creek are in the slate belt of North Carolina where the depth of soil above bedrock is usually only from 0 to 3 ft. Consequently, there is little space for ground-water storage, and runoff is rapid and flashy in character. The low-water

[21] "Studying Sediment Loads in Natural Streams," by Gilbert C. Dobson and Joe W. Johnson, *Civil Engineering*, February, 1940, pp. 93–96.

flows are relatively small; hence, the duration curves for these two streams plot relatively lower than the other streams in Fig. 7. On the other hand, the drainage basins of the East and West forks of Deep River are in the granitic

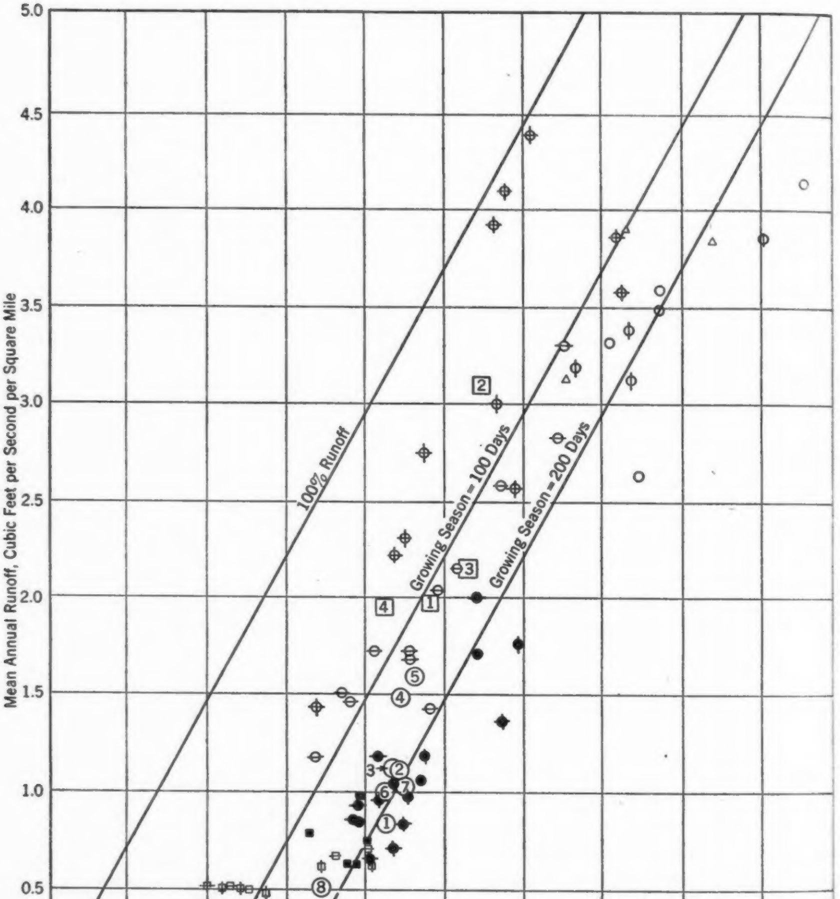

FIG. 8.—RELATION BETWEEN MEAN ANNUAL RUNOFF AND ANNUAL PRECIPITATION FOR STREAMS IN VARIOUS LOCALITIES IN THE UNITED STATES

schist region of the Piedmont where weathering is relatively deep and a considerable depth of soil is available for ground-water storage. Thus, the low-water flows in these streams are relatively high and the part of the duration curves representing these flows plot higher than any of the other curves. It

is a fortunate condition in connection with bed-load problems that the duration curves for streams in a particular region are identical for the higher rates of flow, because it is during these flows that the greatest percentage of the annual volume of bed load is transported. This condition is of particular importance when it is desired to use a duration curve in estimating the bed load of a stream for which no discharge records are available.

For example, on a stream in the North Carolina Piedmont, an average curve may be used in conjunction with a suitable bed-load formula in estimating volumes of bed-load transportation. This average curve (Fig. 7) may not permit an accurate estimate to be made of the load transported during low-water flows, but since little material is transported at such times the errors involved in using the mean curve will be small. To apply the average flow-duration curve to a particular stream, a value for the mean discharge for the stream under consideration must be estimated or assumed so that the ordinates of the curve can be converted to actual discharge. An aid in estimating the mean discharge of a stream is provided by the results of past hydrologic studies which have shown that, in general, the mean annual discharge is related to the annual precipitation.[22] That this relationship is not constant for individual years is evident from Fig. 8, which shows mean annual runoff in cubic feet per second per square mile plotted against annual precipitation in inches for various streams in the North Carolina Piedmont, as well as for streams in Iowa, Wisconsin, Washington, and the mountains of North Carolina. A close examination of Fig. 8, however, reveals a reasonable explanation for the scattering of the individual points, and, with proper consideration of the various factors, a usable relationship between rainfall and runoff can be determined as follows:

If 100% of the annual precipitation appears as runoff, a single straight line (Fig. 8) would describe the relationship between these variables. Because a certain amount of rainfall is lost by evaporation and transpiration, however, the plotted points fall to the right of this 100% line. Previous investigations[23] have shown that curves drawn through data from mountainous regions occupy a different position than that for curves of data from valleys. The main effect of the location of a stream appears to be in the influence of length of growing season on evaporation and transpiration losses. Thus, as a guide to the construction of curves from the data plotted in Fig. 8, a plot was made of the annual water loss (represented by the horizontal distance from the 100% curve to the plotted point) against the average length of growing season for the county, or counties, in which the drainage basin of the stream is located. (Data on the length of the growing season were obtained from the 1941 *Yearbook of Agriculture* in which a climatic summary for practically every county in the United States is given.[24]) A straight line was then drawn through the plotted points. If it is now assumed that a certain base rainfall is required to supply evaporation and transpiration losses, and that practically all

[22] "The Elements of Hydrology," by A. F. Meyer, John Wiley & Sons, Inc., p. 446.

[23] "Results of Stream Measurement," by F. H. Newell, Fourteenth Annual Rept., U. S. Geological Survey, Pt. 2, 1892–1893, pp. 89–155.

[24] "Climate and Weather Data for the United States," *Yearbook of Agriculture*, 1941, U. S. Government Printing Office, Washington, D. C., pp. 685–1228.

precipitation greater than this amount appears as runoff, curves may be constructed as in Fig. 8. Thus, from the plot of water loss against length of growing season, a loss of 20 in. seemed apparent for a growing season of 100 days, and for a growing season of 200 days the loss was approximately 30 in. By drawing lines through the base amounts of 20 in. and 30 in. of rainfall, and parallel to the 100% runoff curve, the curves shown in Fig. 8 were determined. It is to be noted that the curve for a growing season of 100 days passes through the group of points obtained from mountainous regions and from regions in the northern latitudes of the United States. It may be observed further that the curve for a growing season of 200 days passes through the data from streams in the Piedmont and Iowa. As a first approximation, these curves have been constructed as straight lines; however, further study may show that curved lines describe the relationship better.

It is acknowledged that the relationship between runoff and precipitation, as shown in Fig. 8, is merely a generalization that reveals class likenesses and obscures the differences between the individual characteristics of runoff and the character and distribution of rainfall (as well as the effect of temperature, vegetal cover, topography, soil, and subsoil) on the disposal of rainfall. Although the method of arriving at an estimated flow-duration curve for a particular stream upon which all hydrologic data are missing may not be compatible with the results of the more recent hydrologic research, it appears to be the only simple and direct approach to provide the necessary information used in estimating volumes of bed-load transportation.

The use of Figs. 7 and 8 in bed-load studies may be illustrated as follows: Assume that an estimate is desired of the average annual volume of bed load transported in a particular stream in the Piedmont of North Carolina. If no discharge data are available on the stream, the average flow-duration curve from Fig. 7 must be used. The ordinates of this curve are converted to actual discharges by the use of Fig. 8. From the map of average annual rainfall, shown in the 1941 *Yearbook of Agriculture*,[24] the annual rainfall for the drainage basin of the stream is determined. From the same publication, the average length of growing season is also determined. By entering the abscissa scale of Fig. 8 with the annual rainfall and then proceeding upward until the curve representing the average length of growing season is reached (only the 100-day and 200-day curves are shown), the mean annual runoff in cubic feet per second per square mile is read from the scale of ordinates. This value of mean discharge, multiplied by the area of the drainage basin in square miles, gives the estimated mean discharge. Multiplying each value along the ordinate scale of the mean duration curve (Fig. 7) by the value of the mean discharge, a flow-duration curve with ordinates expressed in cubic feet per second now is available for the stream under consideration.

A table that shows the frequency of occurrence of various discharges (distribution curve) can be prepared from the flow-duration curve—that is, the total range in discharge is divided into a convenient number of parts and the percentage of time that the mean discharge in each part prevails is then determined and tabulated. The time that each discharge prevails per year is

computed. The volume of bed load that is transported per unit time for each discharge may be determined either by a suitable bed-load formula (such as the ϕ-ψ function of H. A. Einstein,[25] Assoc. M. Am. Soc. C. E.) or by a relationship between load and discharge established by direct measurement[26] on the stream under investigation. The rate of transportation per unit of time for a particular discharge, multiplied by the total time that the discharge prevails per year, gives the annual volume of bed load. The sum of the amounts for the various discharges gives the average annual volume of sediment transported as bed load.

In connection with the computations for the duration curves shown in Fig. 7, it is of interest to note that, for Muddy Creek and the East and West forks of Deep River, curves were computed directly from the stage recorder charts as well as from the records of mean daily discharge. The procedure of determining, directly from the recorder charts, the percentage of time that a particular discharge is equaled or exceeded gives the correct curve, because instantaneous discharges are used instead of daily averages. A comparison of the curves as calculated by the two methods showed that, for a particular stream, the curve based on instantaneous discharges was practically coincident with the curve based on mean daily discharges for lower rates of flow; was slightly below the mean daily curve for flows in the vicinity of the average discharge; and was above the mean daily curve for the highest discharges. The relative location of the two curves is to be expected because, during periods of low flow, the instantaneous and the mean daily discharges are practically the same, and the duration curves naturally coincide. For the high rates of flow, the highest values of discharge are obtained when instantaneous discharges are considered; hence, the duration curve based on instantaneous discharges will lie above the curve based on mean daily flows. A plot of the two curves on probability paper shows that, for all practical purposes, the curves can be considered to coincide. Because of the ease with which curves may be calculated from tabulations of mean daily discharges, the curves shown in Fig. 7 were computed on that basis.

The duration curves shown in Fig. 7 are based on records which are probably far too short to permit the development of a curve that is typical of the North Carolina Piedmont. In fact, for only a few rivers in the United States is it possible to obtain records for a period sufficiently long to include a reasonably wide range of conditions. The author's analysis of duration curves, especially as to their accuracy and the type of deviations that may be expected, is of particular value in connection with the applications of the duration curves to bed-load studies as illustrated in this discussion. In such an application, where the shape of the curve in the range of the higher discharges is of primary importance, the material presented by the author is of great value.

[25] "Formulas for the Transportation of Bed Load," by H. A. Einstein, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), pp. 561–577.

[26] "The Flow of Water on a Movable Bed," by H. A. Einstein, presented before the 2d Hydraulic Conference, Univ. of Iowa, Iowa City, Iowa, June, 1942.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

EARLY CONTRIBUTIONS TO MISSISSIPPI RIVER HYDROLOGY

Discussion

BY CHARLES SENOUR, M. AM. SOC. C. E.

CHARLES SENOUR,[26] M. AM. SOC. C. E.[26a]—A well-deserved tribute is paid, in this interesting paper, to the early observers of Mississippi River phenomena, whose painstaking care and understanding went far toward compensating for the shortcomings of their comparatively crude equipment.

Table 1 records annual discharges of the Mississippi River at Natchez for most of the period 1818 to 1859, as obtained from *Professional Paper No. 13.*[23] Captain Humphreys and Lieutenant Abbot measured discharges at Natchez in 1851 and 1858 by subsurface floats. A vertical curve of velocities for the station was prepared and the equation determined for converting velocity at 5-ft depth to mean velocity. From these measurements they developed a stage-discharge relationship curve, and, utilizing this in conjunction with monthly mean stages, deduced the mean annual discharges. Mr. Jarvis has supplemented the Humphreys and Abbot data with later observed or derived data and, in Tables 5 and 6, has extended Table 1 to 1939.

To compare actual measured discharges with those indicated by the stage-discharge curve, the writer plotted all observed discharges at Natchez or at stations between that point and Old River from 1851 to date, in terms of the gage used by Humphreys and Abbot. Fig. 6 shows the zone of extreme variations of those plottings, as compared with the rating curve. The discharges were found to fit the curve very well except for the stages below 15 ft. Of course, there is a fairly wide spread (about 140,000 cu ft per sec in the upper portion of the field) between rising stage and falling stage discharge for the same gage height, so that the mean curve, as a rule, would not give the true discharge for any particular date except (perhaps) at the crest of a rise; but in using it to develop an annual mean or total discharge such discrepancies

NOTE.—This paper by C. S. Jarvis, M. Am. Soc. C. E., was published in March, 1942, *Proceedings*. Discussion has appeared in *Proceedings*, as follows: September, 1942, by Messrs. Robert Follansbee, and R. W. Davenport.

[26] Head Engr., Mississippi River Commission, Vicksburg, Miss.

[26a] Received by the Secretary October 1, 1942.

[23] "Report on the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, *Professional Paper No. 13*, Corps of Engrs., U. S. Army, 1876.

would tend to compensate. At this particular station, the stage-discharge relationship appears to have been rather constant for the years in which discharge was measured. Measurements have been far from continuous, however. The range has never been considered a highly desirable one, because of very deep water and large boils which give rise to questions regarding the

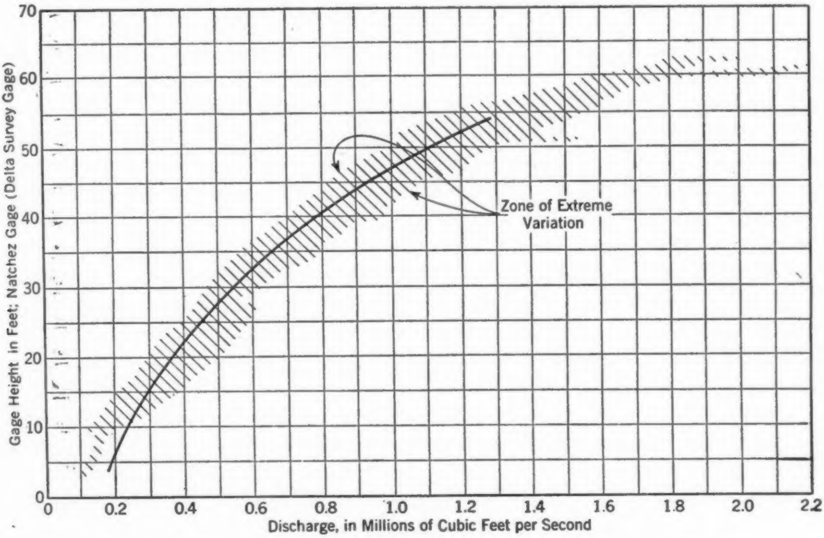

FIG. 6.—DISCHARGE OBSERVATIONS, MISSISSIPPI RIVER, AT NATCHEZ, MISS.

accuracy of both velocity measurements and soundings. Since 1884, measurement of either high water or low water has been made practically every year at both Vicksburg, Miss., and Arkansas City, Ark., and since 1927 the observations have covered a much longer period of each year. At Red River Landing the same process has been in use since 1881. At Natchez, however, following those of Humphreys and Abbot, no measurements were made until 1890–1892. Again there was a hiatus until 1927 and another between 1927 and 1934. In 1911–12–13 measurements were at Point Breeze, about 57 miles below Natchez. In 1928 a range was established at Tarbert Landing, just above Old River, and measurements have been made there annually since that date. Since 1934, annual measurements have also been made at Natchez. What changes the stage-discharge relationship may have undergone during the long periods for which there are no data, and what that relationship may have been between 1818 and the Humphreys and Abbot measurements, are not known. It is known, however that there were major adjustments in the form of Shreves Cutoff and Raccourci Cutoff in 1831 and 1848, respectively.

Constancy of stage-discharge relationship is by no means the rule on the lower river. The gradual growth of the levee system to a stature that really permitted confinement of flood flows to the channel, the gradual lengthening of river bends, and their subsequent elimination from time to time by cutoffs,

natural or planned, together with numerous other phenomena of regimen, all have combined to produce a changing rather than a fixed relationship. At some stations (Arkansas City and Vicksburg, for example) the changes have been very pronounced, as is evidenced by the rating curves for various years submitted by Mr. Jarvis. Records of actual measurements show that at Vicksburg the stage required to pass a rising stage discharge of 1,000,000 cu ft per sec was about 40 ft in 1890, 38 ft in 1896, 40 ft in 1898, 37 ft in 1906, 40 ft in 1916, 42 ft in 1928, and 31 ft in 1940. Bankfull stage is 42 ft. The increment, in discharge per foot of stage, is about 50,000 cu ft per sec on this part of the curve. Deduction of past flows from present rating curves could introduce errors of a couple of hundred thousand cubic feet per second or more in many cases. During periods of Vicksburg observations the results at that point could be translated to Natchez, probably without very serious discrepancy, but this device is not available for the period anterior to Humphreys and Abbot. This is not intended as a condemnation of the methods employed by the author, who has used the data very ingeniously, indeed. Parenthetically it might be observed that at Vicksburg the 1937 rating curve presented by Mr. Jarvis lies about 100,000 cu ft per sec to the left of that developed by the Mississippi River Commission for the same year below the 35-ft stage. There is a closer agreement above that stage.

On the Mississippi River, the derivation of annual discharge by the use of monthly mean stages should yield fairly accurate figures, provided the rating curve reflects the true relationship between discharge and stage and provided no flow by-passes the station by reason of diversions through levee breaks. In extreme floods of earlier years there were numerous crevasses in the levees, and outflow through those in the west bank levees below the Arkansas River would not reenter the Mississippi above Natchez. The method of monthly mean stages was applied to the Vicksburg record for the years 1930 through 1936, using the appropriate rating curve in each case. Compared with the results obtained by adding together the daily discharges as actually measured from day to day or as scaled from loop curves connecting measurements two to several days apart, the mean annual discharge as developed from monthly mean stage was:

Year	Percentage higher or lower than the sum of daily discharges
1930	0.04% higher
1931	2.85% higher
1932	1.03% lower
1933	1.02% lower
1934	0.96% lower
1935	1.61% lower
1936	0.74% lower

In Table 3 Mr. Jarvis presents maximum stage and discharge at Carrollton (New Orleans) for each of the years 1811–1860. The Carrollton gage readings prior to 1847 were apparently deduced from readings on the Natchez gage. The relationship between gages on the lower Mississippi is not constant and the Mississippi River Commission has not as a rule included in its published

pamphlets any stages other than actual gage readings or water marks connected to the gage by levels. Incidentally, crest stages at Carrollton as established by high-water marks and published by the Mississippi River Commission for the years 1832, 1840, and 1844 do not agree with those reported in Table 3. The discharges reported in Table 3 were taken from a Carrollton rating curve developed from discharge measurements at Red River Landing (just below Old River) in 1851 and at New Orleans in 1858. Fig. 7 shows this rating

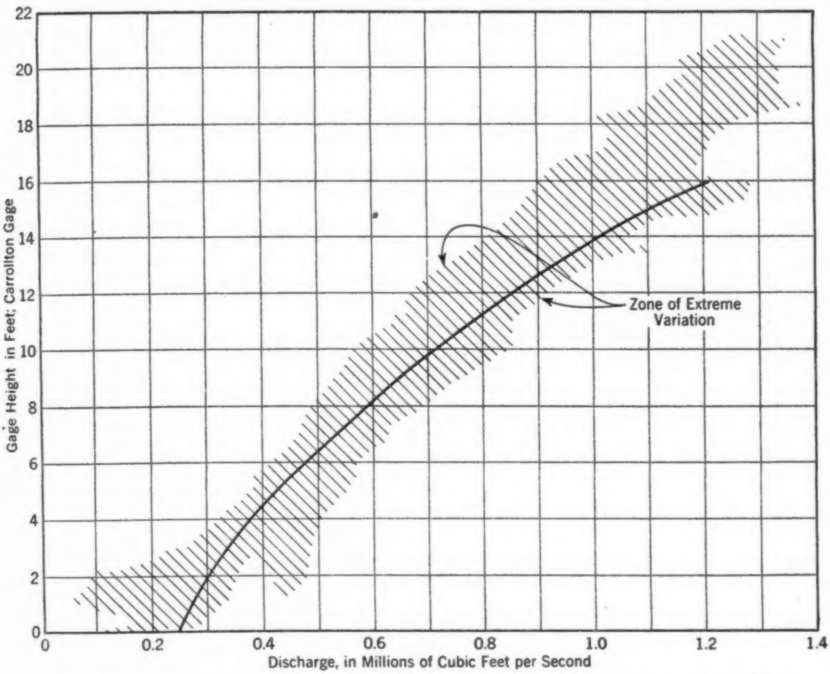

FIG. 7.—DISCHARGE OBSERVATIONS, MISSISSIPPI RIVER, AT CARROLLTON, LA. (1851–1941)

curve about which the writer had plotted all available discharge observations at Carrollton. The zone of extreme variation of these plottings is indicated by the cross-hatched area.

Some clarification of the data presented in Table 7 would be desirable. The table presents maximum gage heights and discharges at Memphis and Vicksburg for the period 1858 to 1937. The Memphis stage for June 23, 1858, should read (with reference to zero of present gage) 34.16 ft instead of 35.3 ft. The stages tabulated for Vicksburg from 1858 to 1931 relate to the U. S. Engineer gage, whereas those subsequent to 1931 apparently relate to the U. S. Weather Bureau bridge gage and are from 0.6 to 2.3 ft lower than the corresponding readings on the Engineer gage. Many of the discharges tabulated are unofficial or estimated quantities. Presumably they are estimated confined flows and so, in the cases of floods not completely confined, do not correspond to the observed gage heights tabulated opposite.

It is noted that for 1909, 1916, 1920, and 1928, estimated discharges are given for Vicksburg, whereas these floods were fully confined at that point and measured confined discharges are available. The discharge given for 1922 was measured after a crevasse had developed 93 miles below Vicksburg and two days after the crest had passed, and was probably influenced by the crevasse. The Vicksburg discharge given for 1927 (namely, 2,495,000 cu ft per sec) appears to be made up of the estimated confined inflow at Arkansas City plus the flow of the Yazoo River. The estimated confined flow at Vicksburg[27] is 2,278,000 cu ft per sec. The actual measured peak flow was 1,806,000 cu ft per sec on May 1, 1927, at a stage of 57.2 ft. The maximum stage was 58.4 ft on May 4, 1927.

The value (2,020,000 cu ft per sec) is the actual measured confined discharge past Memphis in 1937. It is not believed necessary to add the extra 500,000 cu ft per sec.

The data presented in Table 7 should be used with caution. The peak flows of the older great floods were seriously affected by crevasses both above and below the gaging points. The only common basis for comparison is that of confined flow, and it is extremely difficult in many cases to arrive at a satisfactory estimate of it.

In conclusion it might be well to comment on the reference to the blue clay in which the early and many later observers fancied the river bed to lie. Borings disclose that the bed is in sands and gravels. The clays generally exist in old abandoned channels of the river and in old marshes and appear to be the results of deposits of fines from the river itself.

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[27] "Basic Data, Mississippi River," *Annex No. 5*, H.R. Doc. No. 798, 71st Cong., 3d Session, 1931, p. 105.

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DISCUSSIONS

DESIGN OF SIGN LETTER SIZES

Discussion

BY ADOLPHUS MITCHELL, ASSOC. M. AM. SOC. C. E., AND T. W. FORBES, ESQ.

ADOLPHUS MITCHELL,[19] ASSOC. M. AM. SOC. C. E., AND T. W. FORBES,[20] ESQ.[20a]—It was hoped that more discussions would be written; but, since the two published were by men who are leaders in this field, it may be said that the lack of quantity was made up for by the quality.

In closing, it appears to be in order to restate that it was not intended, in the paper, to discuss the attention-getting properties of signs. This element in sign work usually will determine the location of the signs. After the position of each sign has been determined, the sign letter sizes can be designed in accordance with the principles stated in the paper.

The writers wish to thank Messrs. Kelcey and Noble for their fine discussions.

NOTE.—This paper by Adolphus Mitchell, Assoc. M. Am. Soc. C. E., and T. W. Forbes, Esq., was published in January, 1942, *Proceedings*. Discussion has appeared in *Proceedings*, as follows: May, 1942, by Guy Kelcey, M. Am. Soc. C. E.; and June, 1942, by Charles M. Noble, M. Am. Soc. C. E.

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[20a] Received by the Secretary October 21, 1942.